EE 435

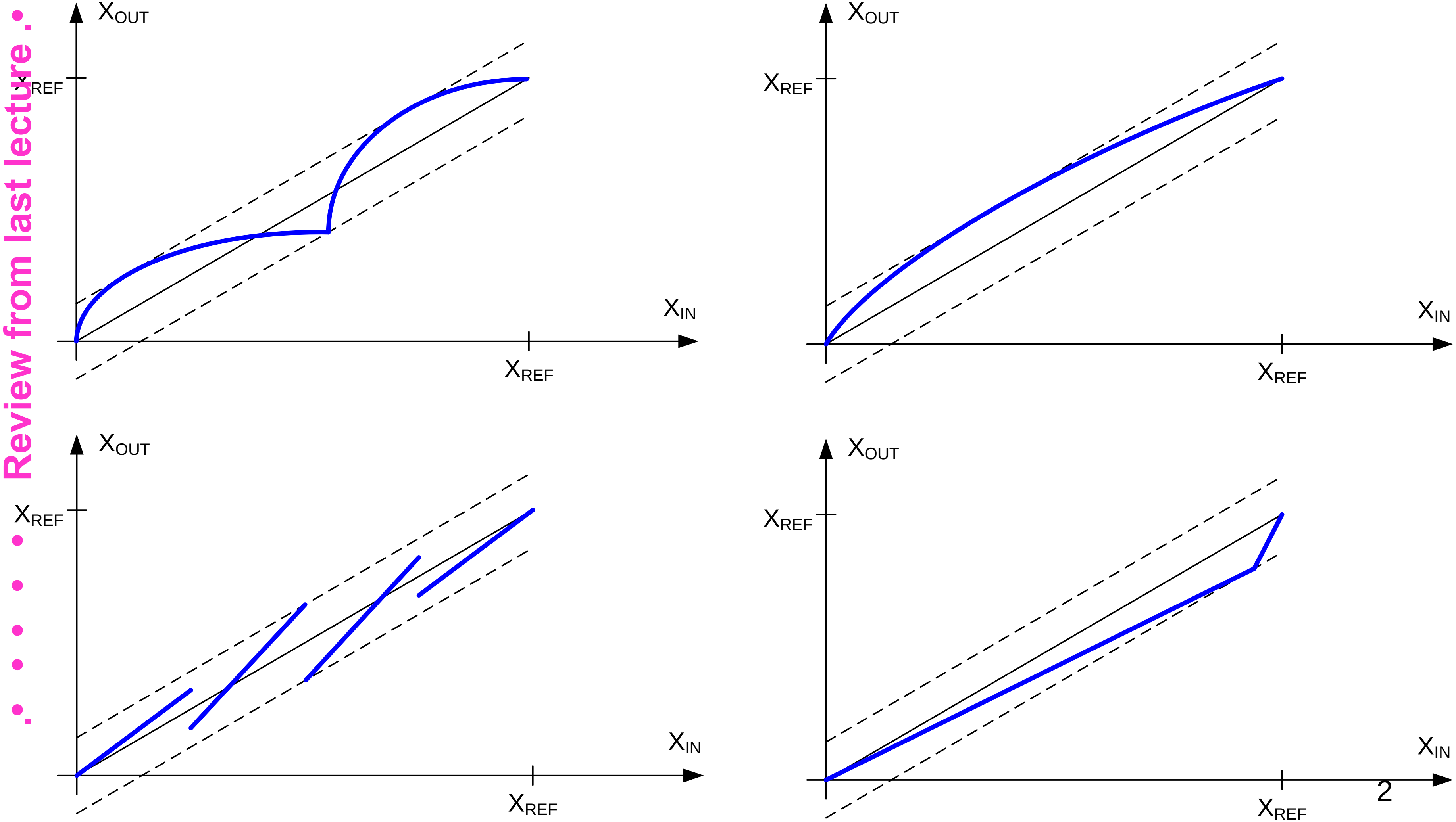
Lecture 28

Data Converters

- Spectral Performance

INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity



Review from last lecture

Linearity Issues

Review from last lecture

- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform

Distortion Analysis

Review from last lecture

Total Harmonic Distortion, THD

$$\mathrm{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}}$$

$$\mathrm{THD} = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}}{\frac{A_1}{\sqrt{2}}}$$

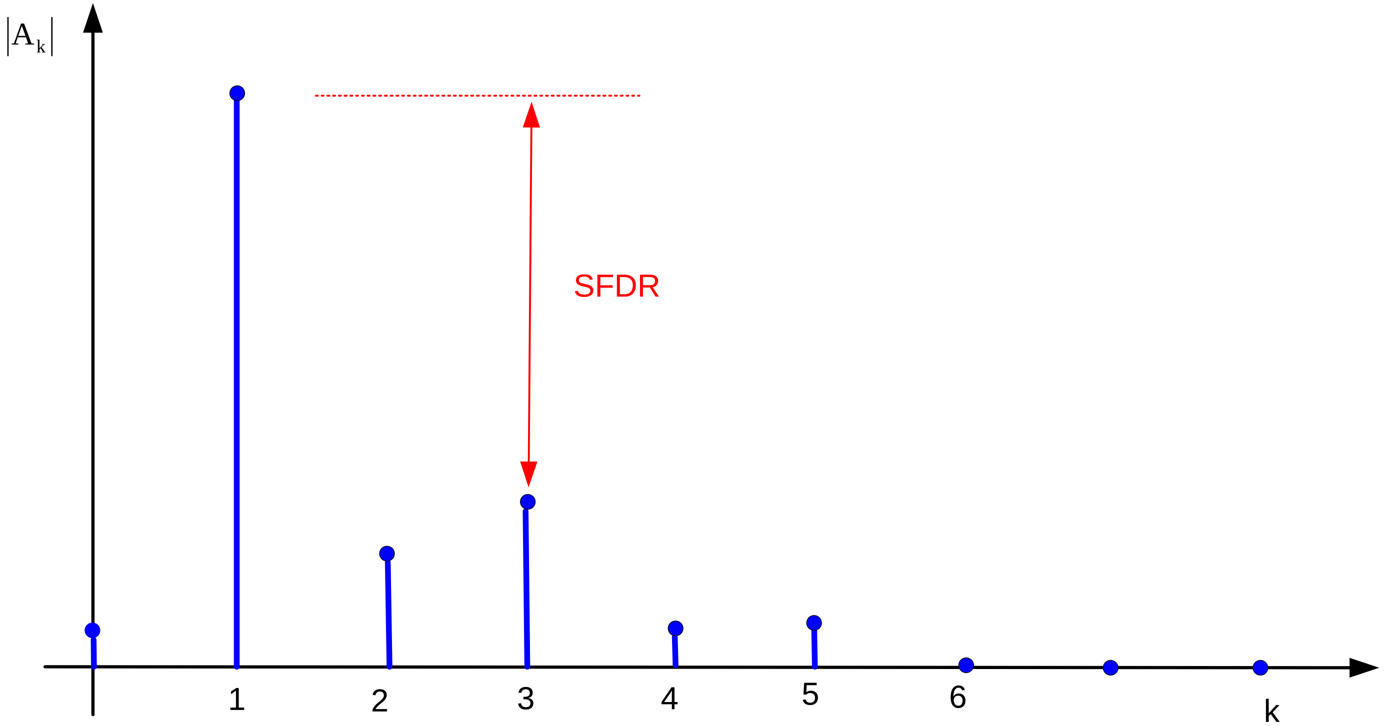
$$\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1}$$

Distortion Analysis

Review from last lecture .

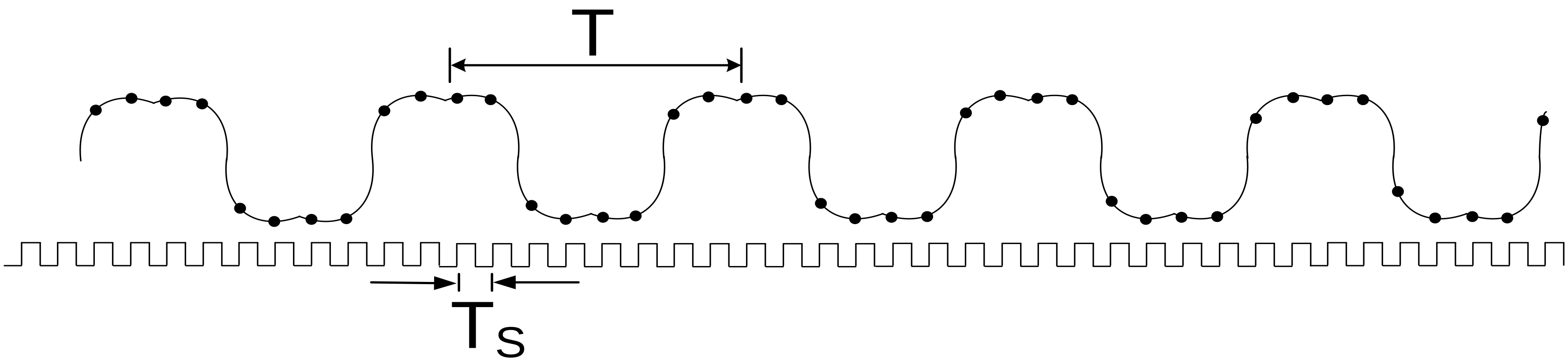
Spurious Free Dynamic Range, SFDR

The SFDR is the difference between the fundamental and the largest harmonic



SFDR is usually determined by either the second or third harmonic

Distortion Analysis

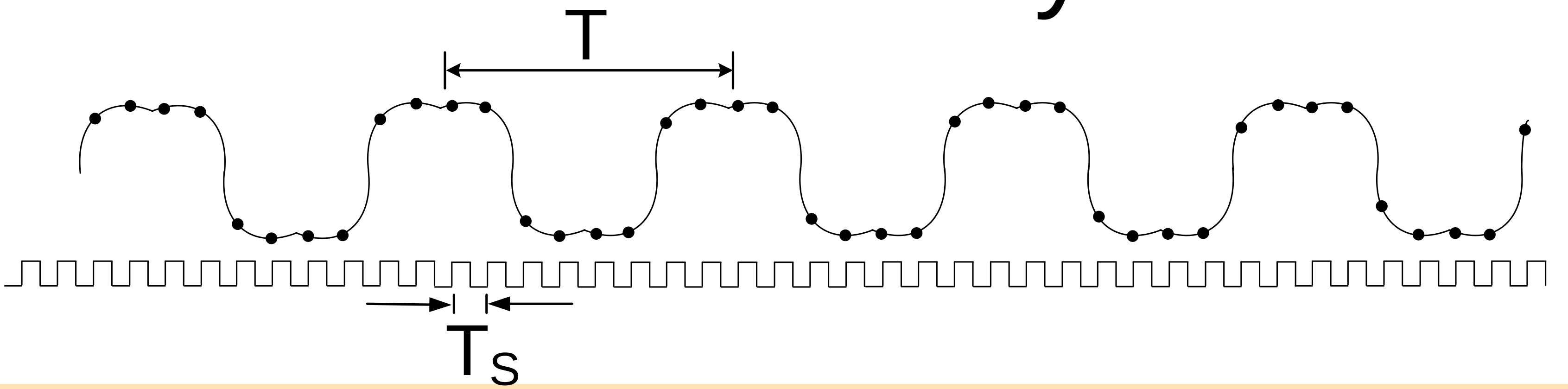


Review from last lecture . . . . . .

THEOREM (conceptual) : If a band-limited periodic signal is sampled at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the DFT of a sampled sequence.

$$\left\langle x(kT_S)\right\rangle_{k=0}^{N-1} \Longleftrightarrow \left\langle X(k)\right\rangle_{k=0}^{N-1}$$

Distortion Analysis



THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_S=1/f_S$. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

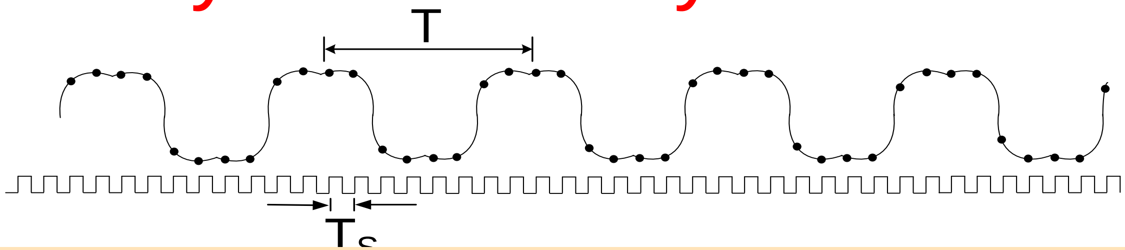
where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

Note spectral components of interest as mN_p+1

Key Theorem central to Spectral Analysis that is widely used !!! and often “abused”

Why is this a Key Theorem?



Review from last lecture

THEOREM: Consider a periodic signal with period T=1/f and sampling period T_S=1/f_S. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

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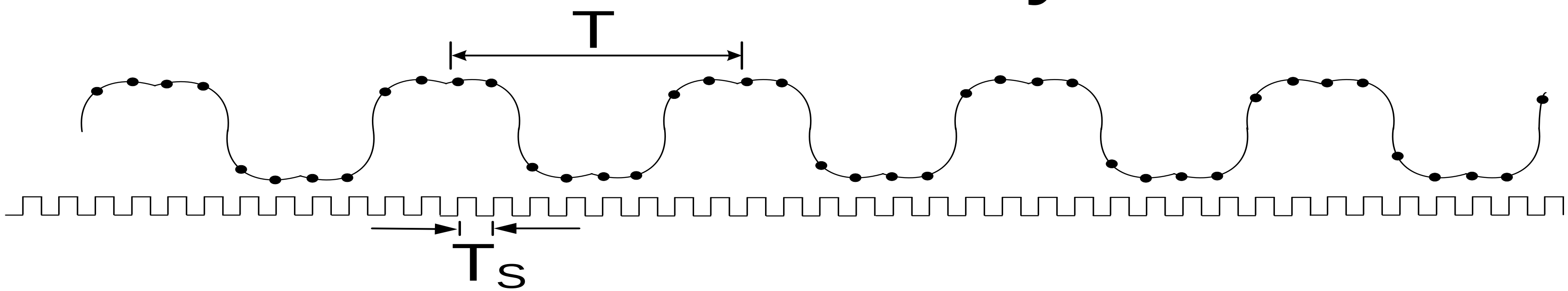
N=number of samples, N_P is the number of periods, and h = $\text{Int}\left(\frac{f_{MAX}}{f}-\frac{1}{N_P}\right)$

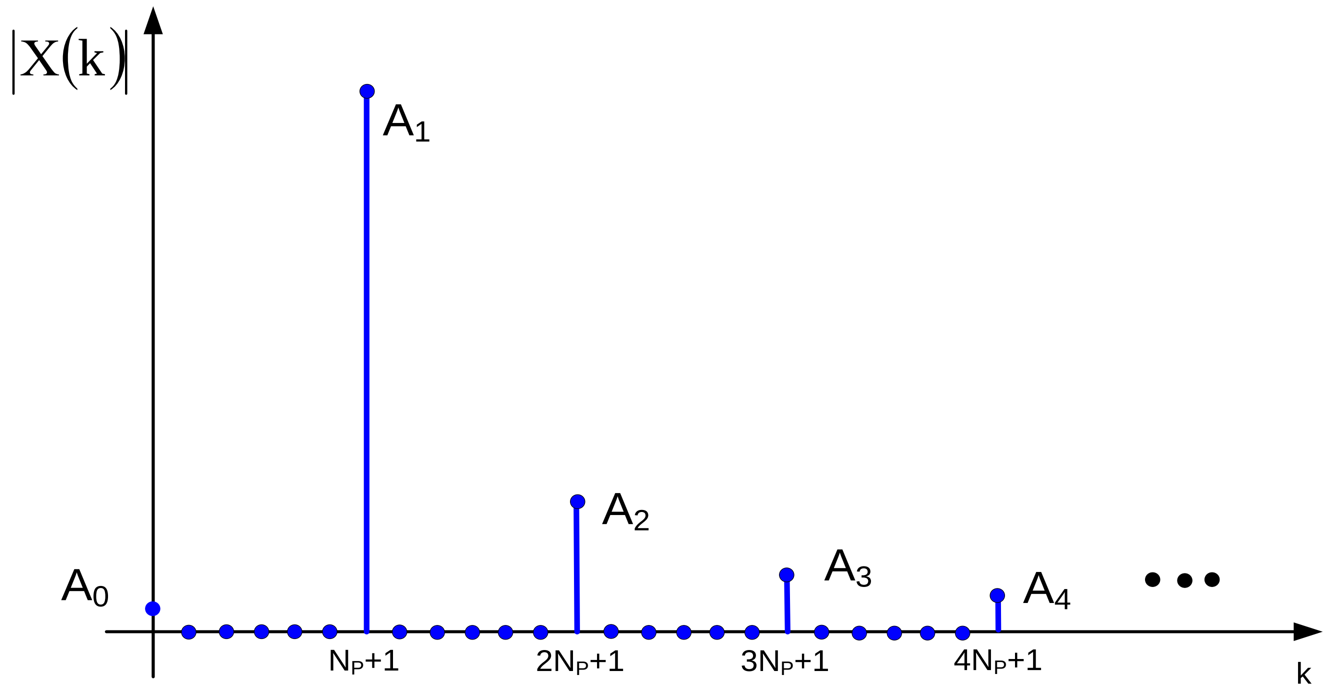
- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem

Review from last lecture

Distortion Analysis



If the hypothesis of the theorem are satisfied, we thus have



Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Review from last lecture .

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

$$f_{MAX\text{-}ACT} = 100\text{Hz}$$

Consider N_P=20 N=512

$$f_{MAX} = \frac{f_{SIG}}{2} \bullet \left[\frac{N}{N_P}\right] = \frac{50}{2} \bullet \frac{512}{20} = 640\,Hz$$

$$f_{MAX\text{-}ACT} << f_{MAX}$$

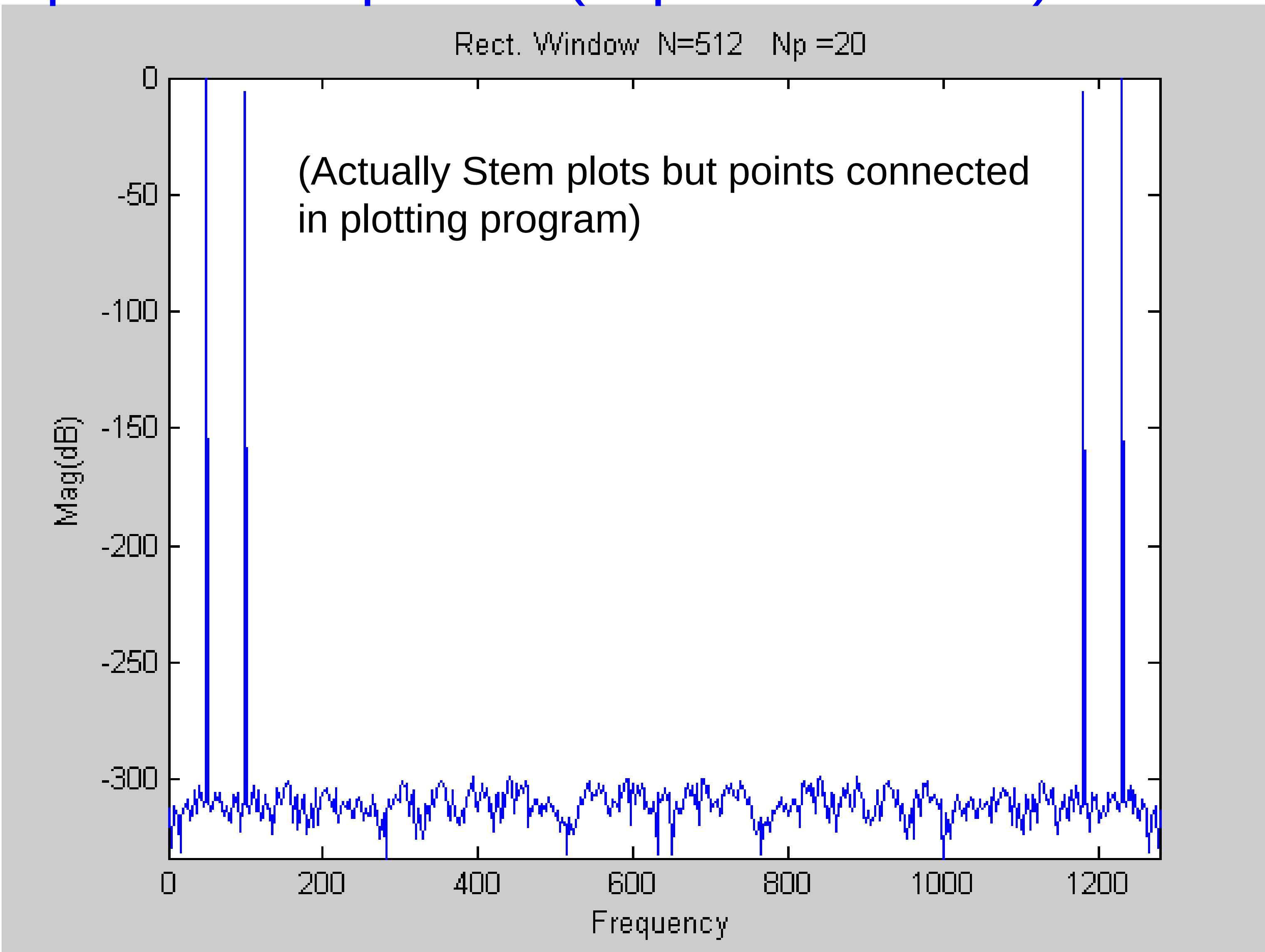
$$f_{SAMPLE} = \frac{1}{T_{SAMPLE}} = \frac{1}{\left(\frac{N_P \bullet T_{SIG}}{N}\right)} = \left[\frac{N}{N_P}\right] f_{SIG} = 2f_{MAX} = 1280\,Hz$$

Recall $20\log_{10}(1.0)=0.0000000$

Recall $20\log_{10}(0.5)=-6.0205999$

Review from last lecture

Spectral Response (expressed in dB)



(Horizontal axis is the “Index” axis but converted to frequency)

$$f_{AXIS} = f_{SIGNAL}\frac{n-1}{N_P}$$

Fundamental will appear at position 1+Np = 21

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15

-308.2339 -317.7064 -315.3135 -307.9349 -304.5641

Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

-0.0000 -307.7012 -312.9902 -312.8737 -305.4320

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db

Second Harmonic at 1+2Np = 41

Columns 26 through 30

-307.8301 -309.0737 -305.8503 -312.2772 -315.7544

Columns 31 through 35

-311.9316 -316.0581 -318.3454 -306.4977 -308.6679

Columns 36 through 40

-309.9702 -305.9809 -322.1270 -310.6723 -310.3506

Columns 41 through 45

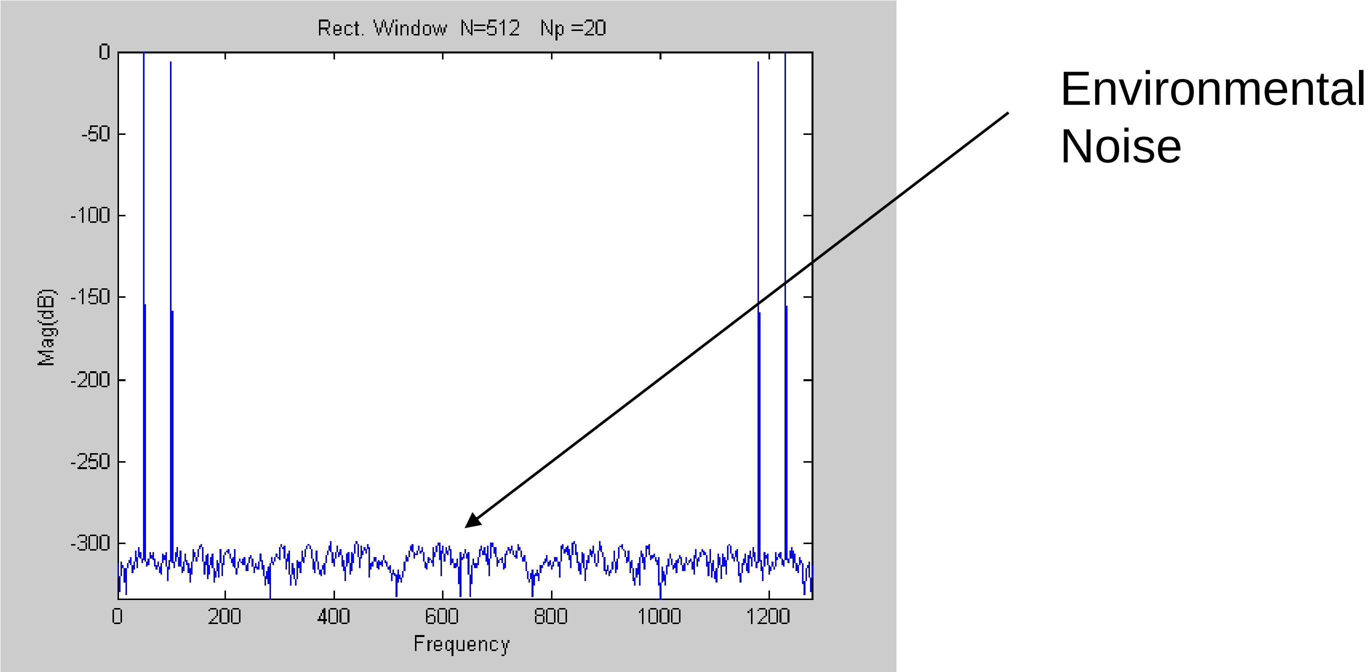
-6.0206 -309.6071 -314.1026 -307.6405 -302.9277

Columns 46 through 50

-313.0745 -304.2330 -310.8487 -317.7966 -316.3385

.• • • • • Review from last lecture .• • • • •

Question: How much noise is in the computational environment?



Observation: This noise is nearly uniformly distributed

The level of this noise at each component is around -310dB

Question: How much noise is in the computational environment?

Assume A_k = -310 dB for $0 \le k \le N$

$$A_{kdB} = 20\log10 A_k \quad \Rightarrow \quad A_k = 10^{\frac{A_{kDB}}{20}}$$

$$A_k \cong 10^{\frac{-310}{20}} = 10^{-15.5} \overset{defn}{=} \bar{A}$$

$$V_{Noise,RMS} \cong \sqrt{\sum_{k=1}^{N-1}\left(\frac{A_k}{\sqrt{2}}\right)^2} \underset{=}{\overset{A_k=\bar{A}}{N \text{ large}}} \bar{A}\sqrt{\frac{N}{2}}$$

$$V_{Noise,RMS} \cong \bar{A}\sqrt{\frac{N}{2}} = 10^{-15.5}\sqrt{\frac{512}{2}} = 5.1 \bullet 10^{-15} \cong 5\text{fV}$$

This computational environment has a very low total computational noise and does not become significant until the 46-bit resolution level is reached !!

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Example - Increase DFT length from 512 to 4096

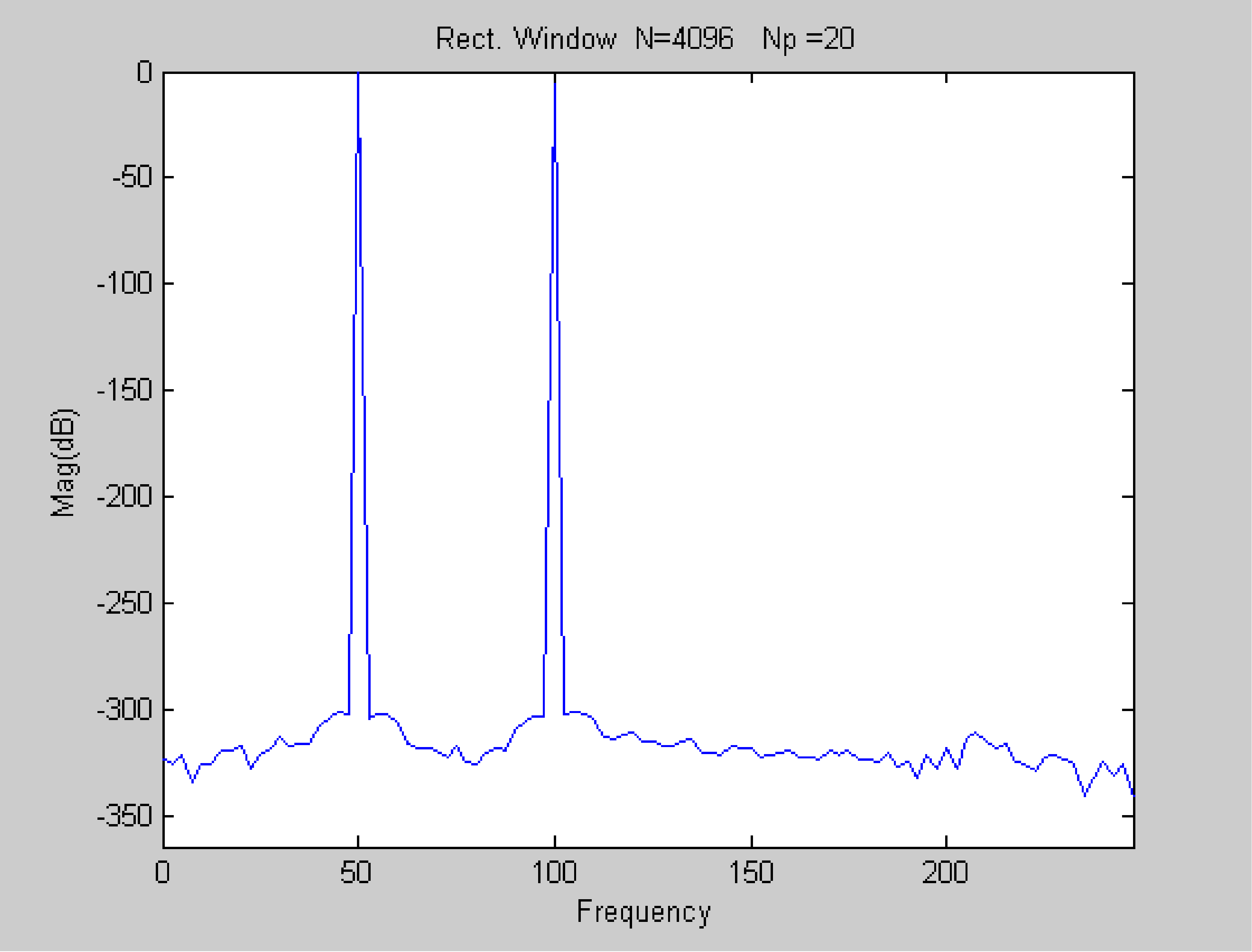
WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20 N=4096

Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-323.9398 -325.5694 -321.3915 -334.6680 -325.2463 -325.3391 -319.3569

Columns 8 through 14

-319.7032 -317.4419 -327.4933 -321.1968 -318.2241 -312.7300 -316.8359

Columns 15 through 21

-315.5166 -316.1801 -307.8072 -304.3414 -301.3326 -301.7993 0

Columns 22 through 28

-303.9863 -302.2114 -302.5485 -306.5542 -315.4995 -318.3911 -318.4441

Columns 29 through 35

-318.7570 -322.6054 -317.3667 -324.0324 -325.8546 -320.3611 -317.8960

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 36 through 42

-319.0051 -309.4219 -305.5698 -302.8625 -303.2207 -6.0206 -302.3437

Columns 43 through 49

-300.8222 -301.6722 -304.8150 -313.0288 -313.5963 -312.1136 -310.7740

Columns 50 through 56

-314.7706 -315.3607 -317.0331 -316.8648 -314.4965 -314.3096 -320.4308

Columns 57 through 63

-320.2843 -320.9910 -316.8320 -318.3531 -318.4341 -322.1619 -321.6183

Columns 64 through 70

-320.6985 -319.0630 -322.1485 -322.3338 -323.6365 -319.0865 -321.0791

Example - Increase NP from 20 to 50

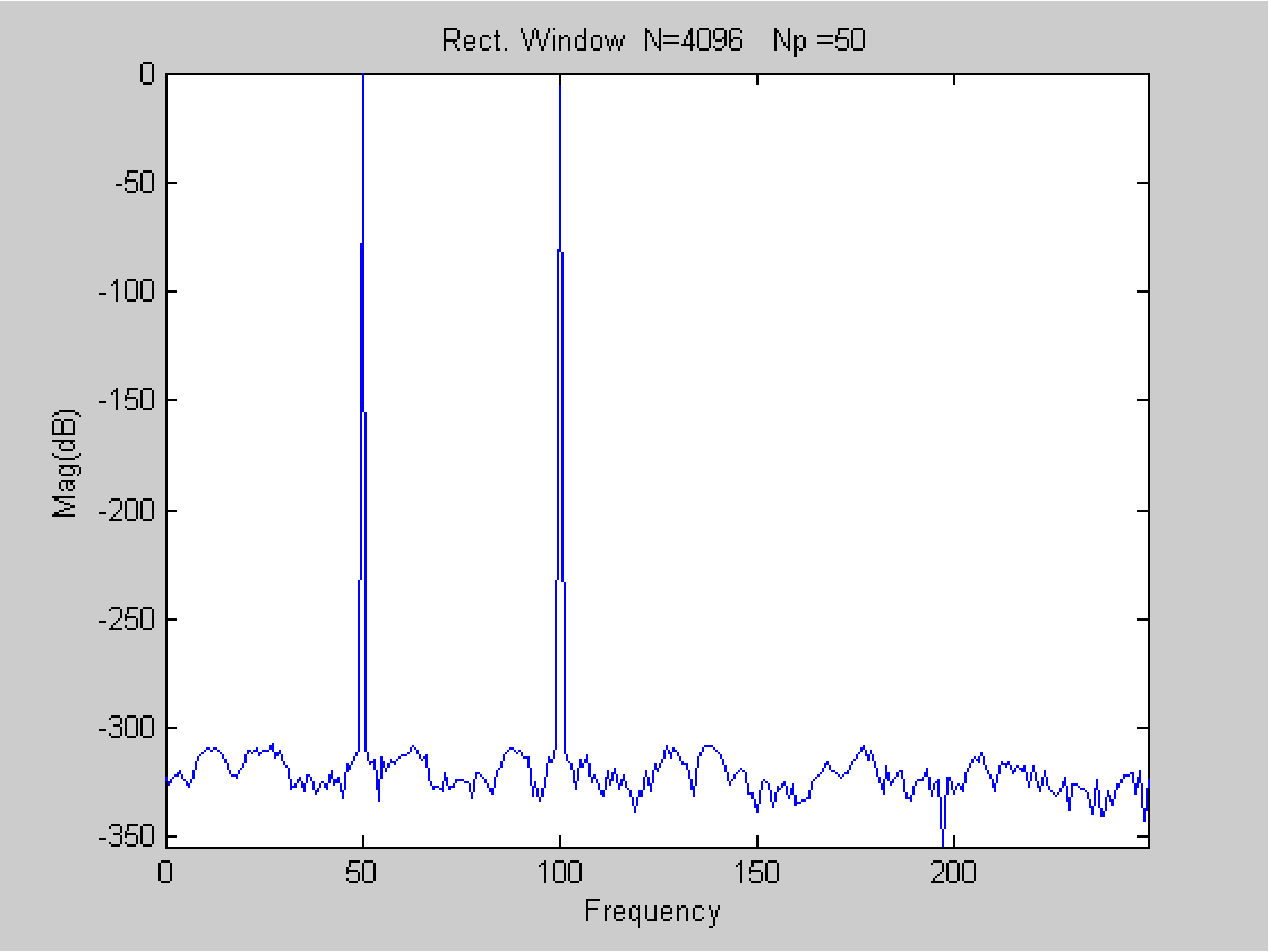
WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=50 N=4096

Spectral Response



Fundamental will appear at position 1+Np = 51

Columns 1 through 7

-322.4309 -325.5445 -322.2645 -321.6226 -319.5894 -323.4895 -327.3216

Columns 8 through 14

-321.2981 -316.1855 -312.3071 -310.4889 -309.6790 -309.9436 -309.3734

Columns 15 through 21

-311.4435 -314.7665 -317.1248 -321.7733 -323.0602 -318.2119 -317.4601

Columns 22 through 28

-310.1735 -311.1633 -308.9079 -312.0709 -310.6683 -310.6908 -307.6761

Columns 29 through 35

-312.9440 -310.5706 -316.2098 -318.9565 -327.6885 -326.4021 -322.3135

Fundamental will appear at position 1+Np = 51

Columns 36 through 42

-328.5059 -321.5592 -322.6183 -330.2002 -328.5051 -324.3480 -328.0173

Columns 43 through 49

-319.3974 -325.8498 -323.1539 -331.9531 -317.0166 -318.3041 -314.9011

Columns 50 through 56

-309.5231 0 -308.8842 -316.1343 -314.5406 -333.4024 -313.7342

Columns 57 through 63

-319.6023 -314.9029 -316.6932 -314.7123 -311.9567 -312.0200 -309.8825

Columns 64 through 70

-308.7103 -309.8064 -314.9393 -312.4610 -322.7229 -328.0350 -326.6767

k^{th} harmonic will appear at position $1+k \bullet Np$

Columns 71 through 77

-329.1687 -321.1102 -328.3790 -326.9774 -323.4227 -323.3388 -325.1652

Columns 78 through 84

-325.3417 -332.1905 -320.4431 -322.1461 -323.8993 -325.4370 -329.8160

Columns 85 through 91

-319.1702 -317.1792 -312.4734 -310.2585 -309.5426 -310.8963 -310.6955

Columns 92 through 98

-313.6855 -313.3882 -330.4962 -324.4762 -333.2237 -325.8694 -313.9127

Columns 99 through 105

-315.4869 -308.6364 -6.0206 -309.2723 -314.4098 -316.3311 -328.2626

k^{th} harmonic will appear at position 1+k•Np

Columns 106 through 112

-314.3378 -317.7599 -312.1738 -324.4699 -321.7568 -326.3796 -331.0818

Columns 113 through 119

-319.9292 -325.4840 -318.0998 -328.0000 -321.7632 -326.5097 -328.5867

Columns 120 through 126

-338.0360 -328.6163 -330.5881 -319.7260 -329.2289 -316.3840 -319.1143

Columns 127 through 133

-315.0684 -308.6315 -312.9640 -309.5056 -311.6251 -316.1369 -316.1064

Columns 134 through 140

-320.4989 -331.2686 -314.3479 -310.0891 -308.0023 -308.1556 -309.0616

k^{th} harmonic will appear at position 1+k•Np

Columns 141 through 147

-311.2372 -312.6180 -319.0565 -325.6750 -323.7759 -320.7444 -318.0752

Columns 148 through 154

-320.5965 -330.3083 -330.2507 -338.2118 -325.0839 -323.5993 -326.2350

Columns 155 through 161

-336.0163 -326.5945 -327.9587 -324.7636 -332.5650 -326.1828 -334.9208

Columns 162 through 168

-333.9169 -333.3995 -332.0925 -324.3599 -322.9393 -320.4507 -317.7706

Columns 169 through 175

-315.9825 -319.2534 -320.8277 -322.3018 -321.6497 -320.4065 -315.4057

Considerations for Spectral Characterization

Quantization Noise

It will be shown that the quantization that takes place in either an ADC or a DAC acts like noise and is nearly uniformly distributed in all DFT bins.

Thus the deviations in output of data converters caused by magnitude quantization is termed quantization noise

It will be shown later that the RMS value of the quantization noise is given by the expression

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Quantization noise components in DFT bins are much larger than the computational noise which is also nearly uniformly distributed in all DFT bins

Considerations for Spectral Characterization

DFT Length and NP

- DFT Length and NP do not affect the computational noise floor
- Although not shown here yet, DFT length does reduce the quantization noise floor coefficients but not total quantization noise

If we assume E_{QUANT} is fixed and no signal present

$$E_{QUANT} \cong \sqrt{\sum_{k=1}^{2^{n_{DFT}}} A_k^2}$$

(these are now the DFT coefficients due to quantization noise, not computation noise)

If the A_k's are constant and equal

$$E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, obtain

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

(this expression is actually independent of input waveform)

Considerations for Spectral Characterization

DFT Length

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Substituting for E_{QUANT}, obtain

$$A_k \cong \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1} 2^{n_{DFT}/2}}$$

This value for A_k thus decreases with the length of the DFT sampline window

Example: if n=16, n_{DFT}=12 (4096 pt transform), and X_{REF}=1V,
then A_k=6.9E-8V (-143dB),

(Note A_k>> computational noise floor (-310dB for Matlab) for all practical n, n_{DFT})

Considerations for Spectral Characterization

FFT Length

If a periodic signal is present, there will be a few spectral terms in the DFT that represent the signal energy. If coherently sampled, these terms will be at the fundamental frequency and harmonics of the fundamental.
Each term in the DFT will include any harmonic content along with a quantization noise component but at the fundamental and harmonics the DFT terms will usually completely dominate the quantization noise part

Under these assumptions, if the A_k's due to quantization noise are constant and equal

$$E_{QUANT} \cong \sqrt{\sum_{\substack{k=1 \\ k \neq harmonic}}^{2^{n_{DFT}}} A_k^2} \Longrightarrow E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, we still obtain for DFT terms that are not harmonic terms

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window must be an integral number of periods
2. $N > \frac{2 f_{max}}{f_{SIGNAL}} N_P$

Example

WLOG assume f_{SIG}=50Hz

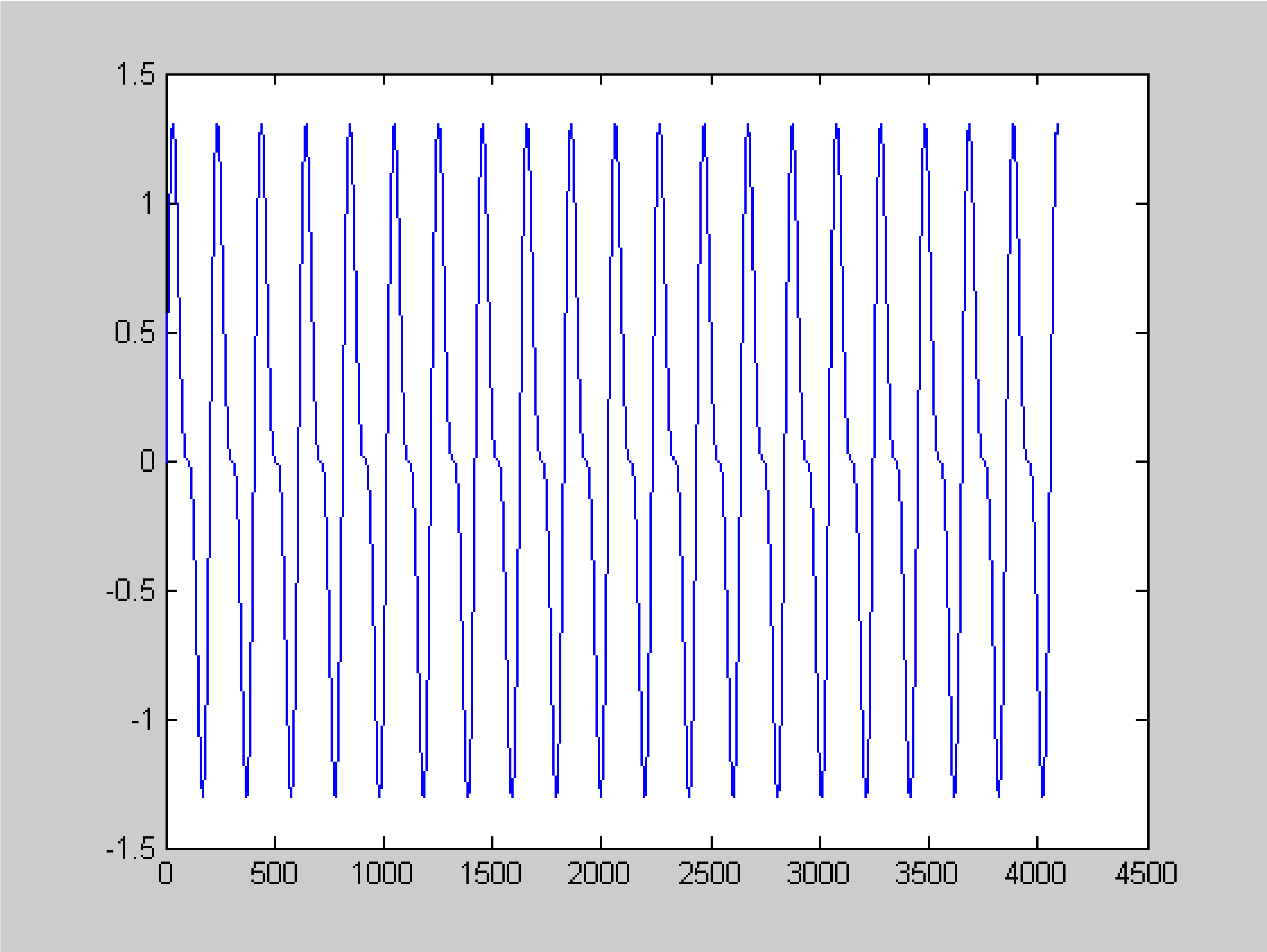
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

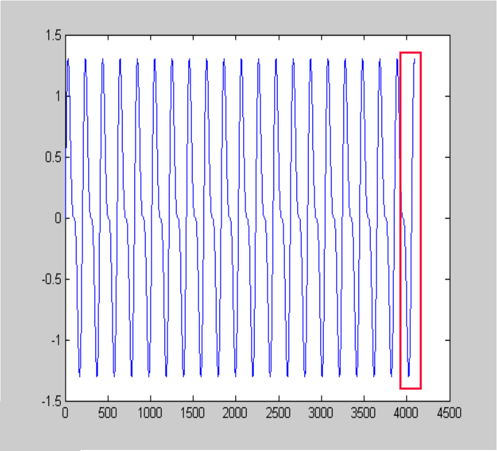
Consider N_P=20.2 N=4096

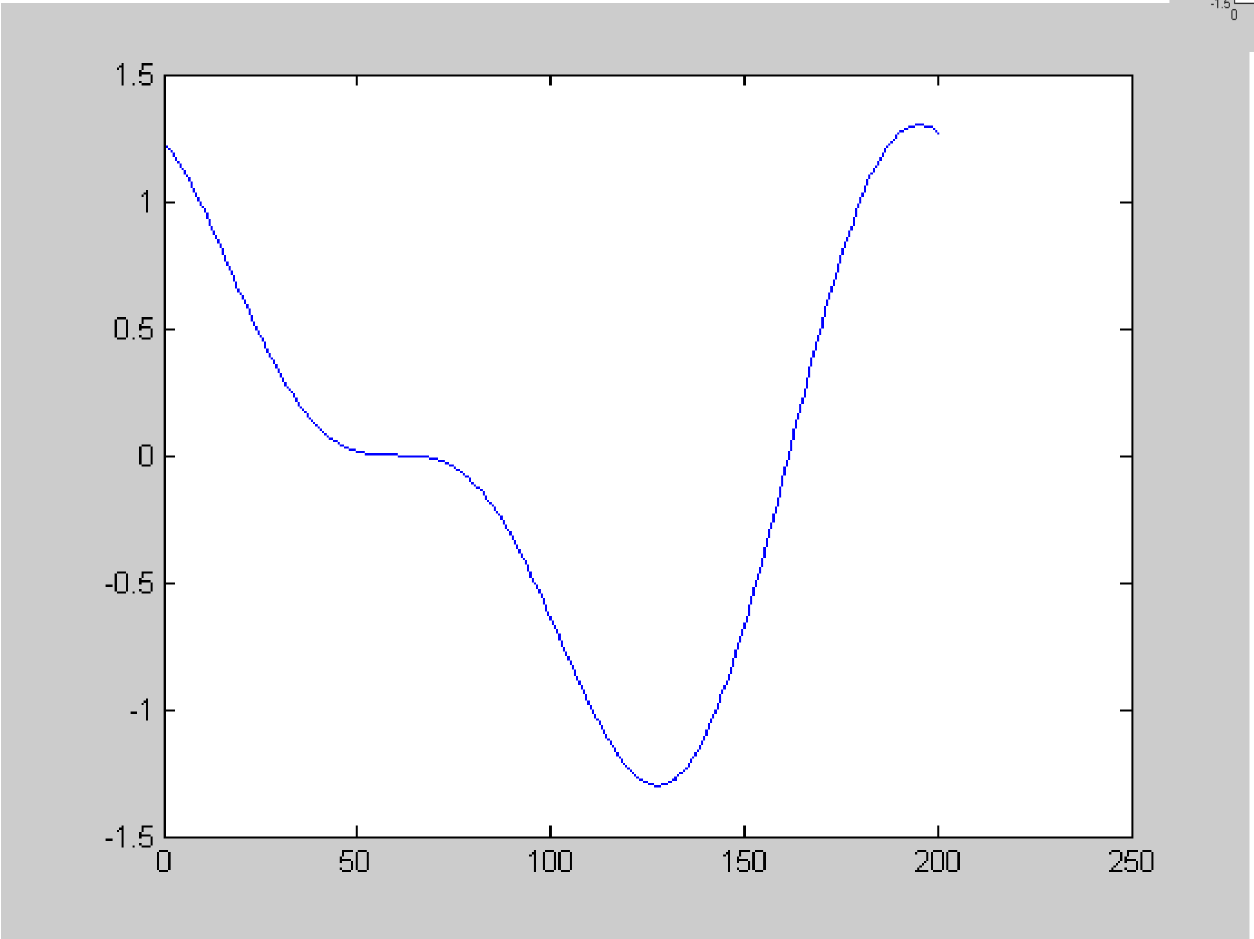
Recall 20$\log_{10}$(0.5)=-6.0205999

Input Waveform

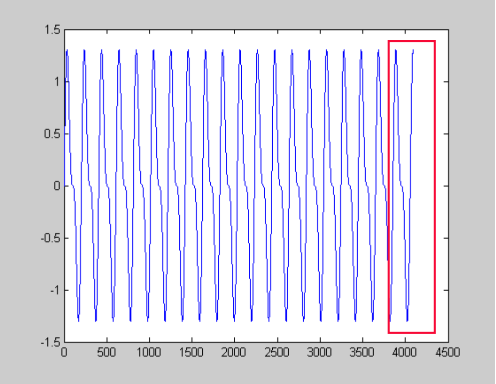


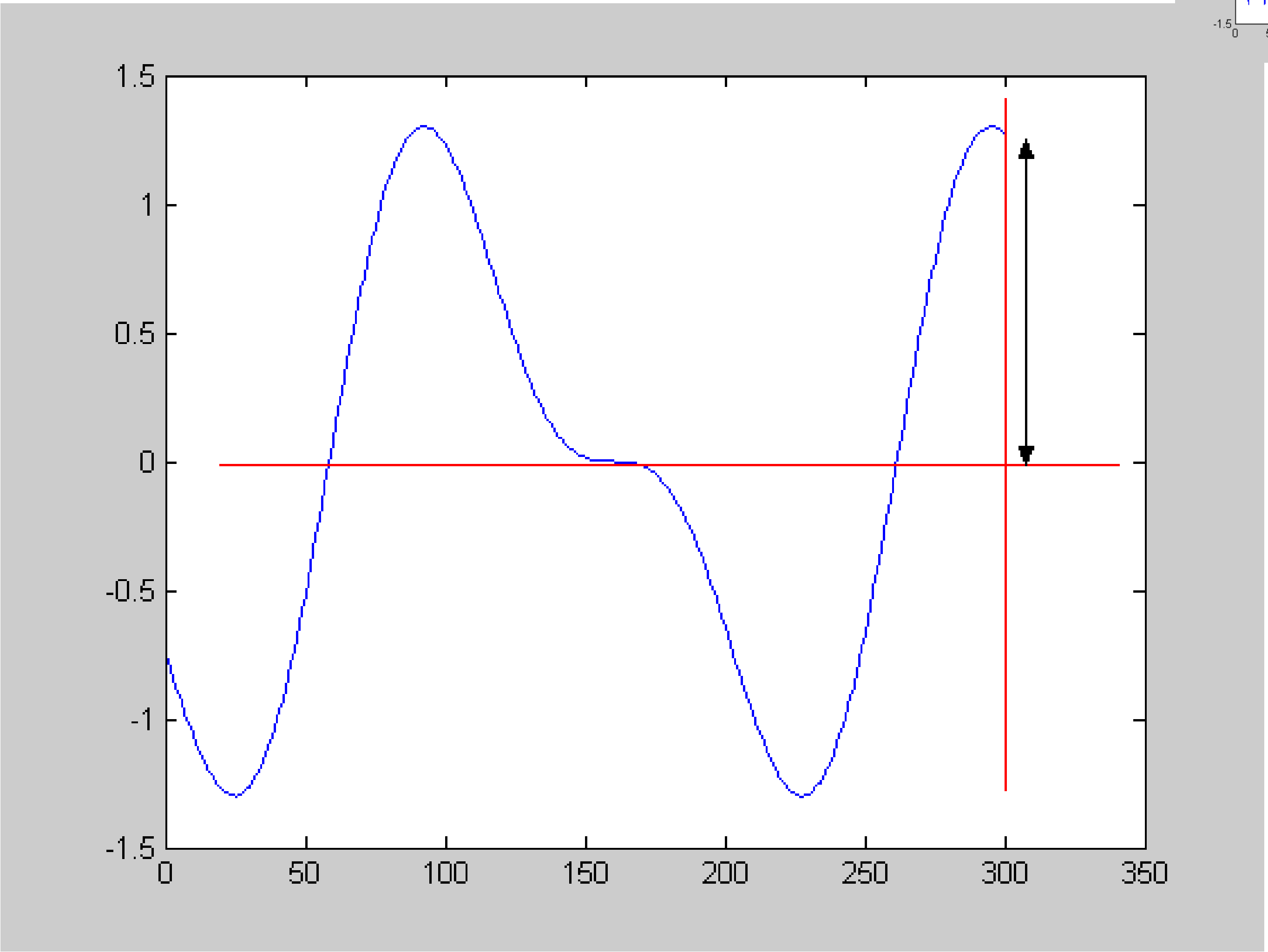
Input Waveform



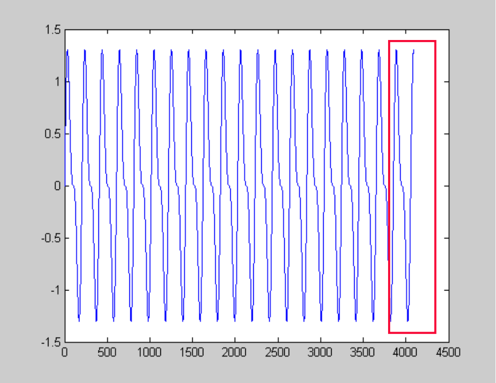


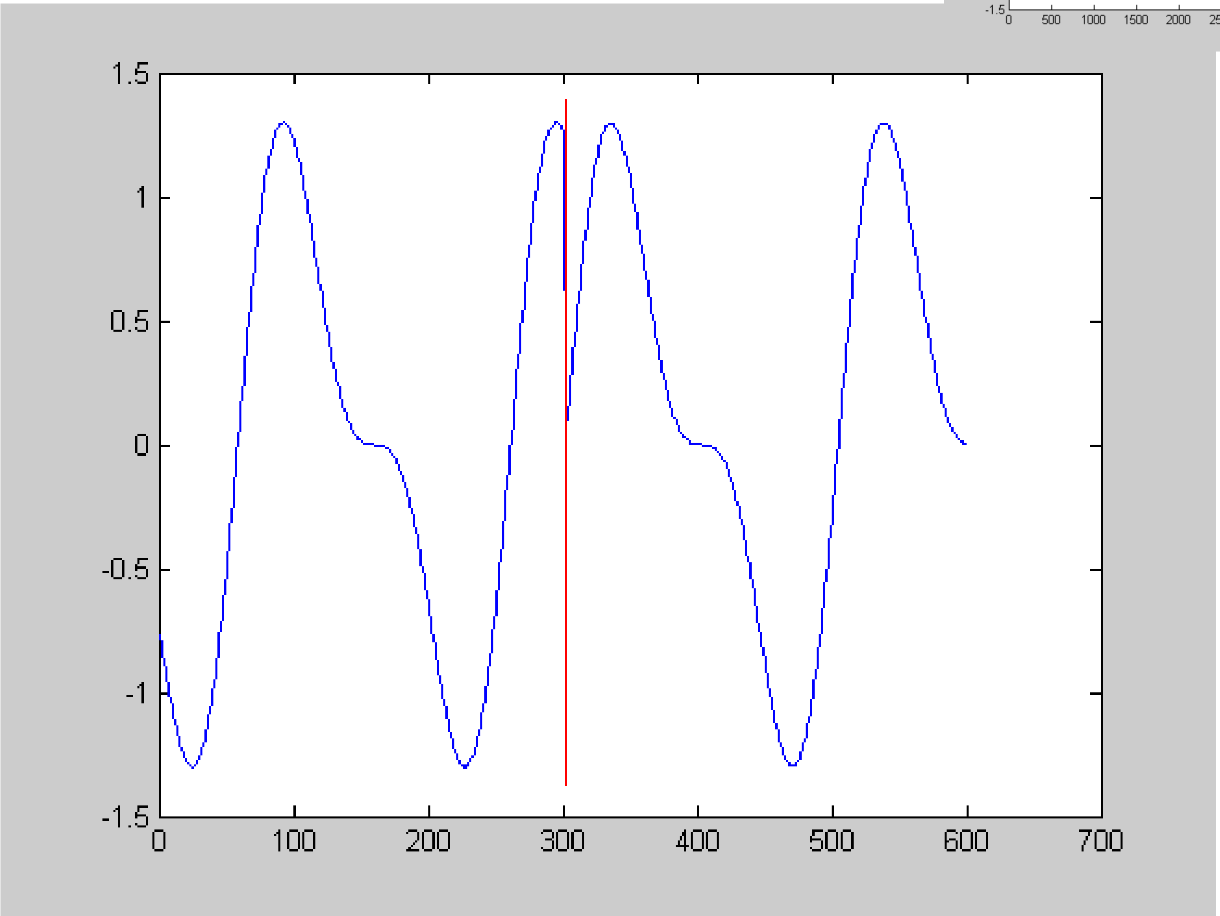
Input Waveform



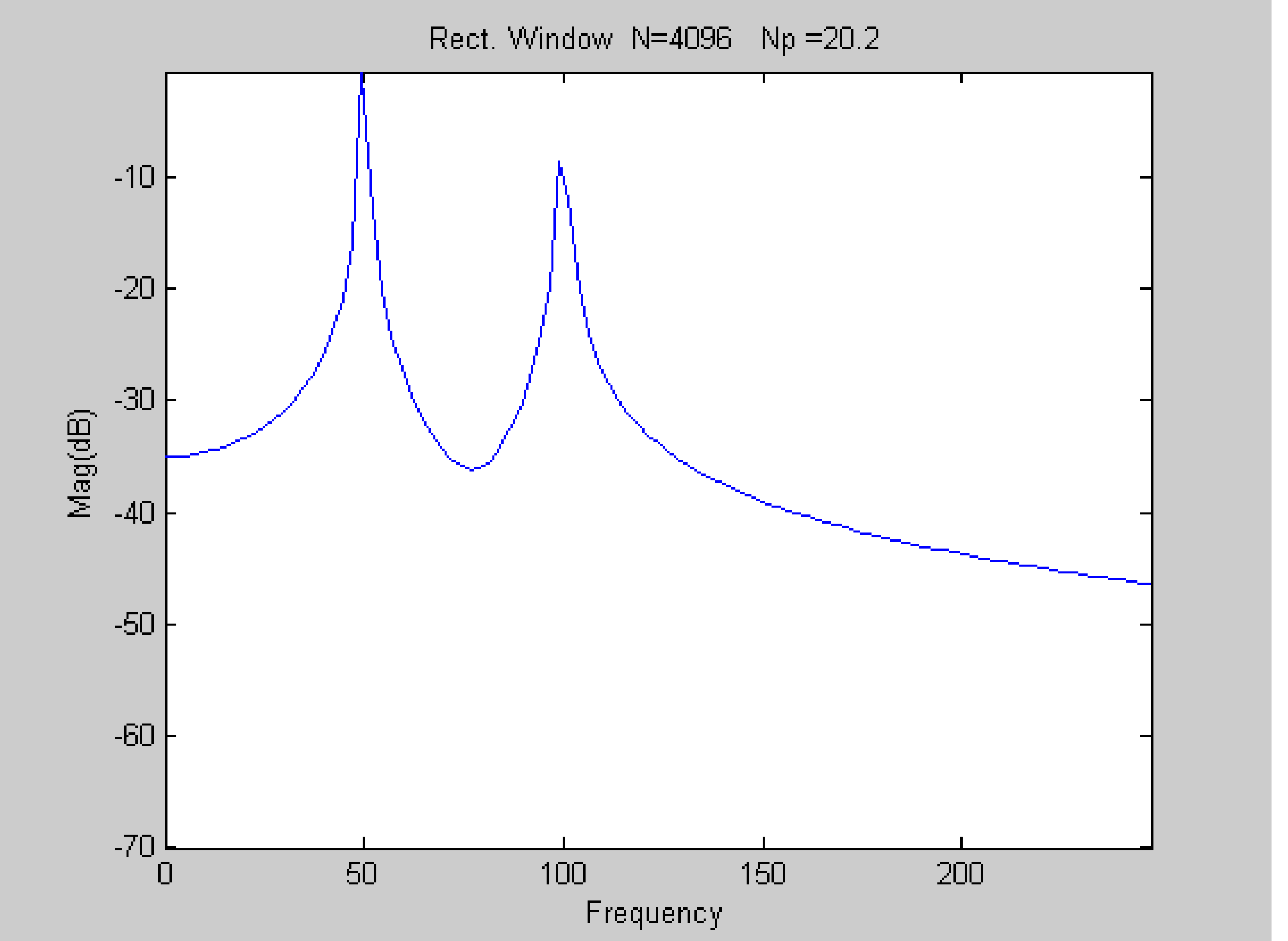


Input Waveform





Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-35.0366 -35.0125 -34.9400 -34.8182 -34.6458 -34.4208 -34.1403

Columns 8 through 14

-33.8005 -33.3963 -32.9206 -32.3642 -31.7144 -30.9535 -30.0563

Columns 15 through 21

-28.9855 -27.6830 -26.0523 -23.9155 -20.8888 -15.8561 **-0.5309**

Columns 22 through 28

-12.8167 -20.1124 -24.2085 -27.1229 -29.4104 -31.2957 -32.8782

Columns 29 through 35

-34.1902 -35.2163 -35.9043 -36.1838 -35.9965 -35.3255 -34.1946

Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350	-30.6397	-28.1125	-24.7689	-19.7626	-8.5639	-11.7825

Columns 43 through 49

-20.0158	-23.9648	-26.5412	-28.4370	-29.9279	-31.1519	-32.1874

Columns 50 through 56

-33.0833	-33.8720	-34.5759	-35.2113	-35.7902	-36.3218	-36.8133

Columns 57 through 63

-37.2703	-37.6974	-38.0984	-38.4762	-38.8336	-39.1725	-39.4949

Columns 64 through 70

-39.8024	-40.0963	-40.3778	-40.6479	-40.9076	-41.1576	-41.3987

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350 -30.6397 -28.1125 -24.7689 -19.7626 -8.5639 -11.7825

Columns 43 through 49

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Columns 50 through 56

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Columns 57 through 63

-37.2703 -37.6974 -38.0984 -38.4762 -38.8336 -39.1725 -39.4949

Columns 64 through 70

-39.8024 -40.0963 -40.3778 -40.6479 -40.9076 -41.1576 -41.3987

Observations

- Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic
- More importantly, dramatic raise in the noise floor !!! (from over -300dB to only -12dB)

Example

WLOG assume f_{SIG}=50Hz

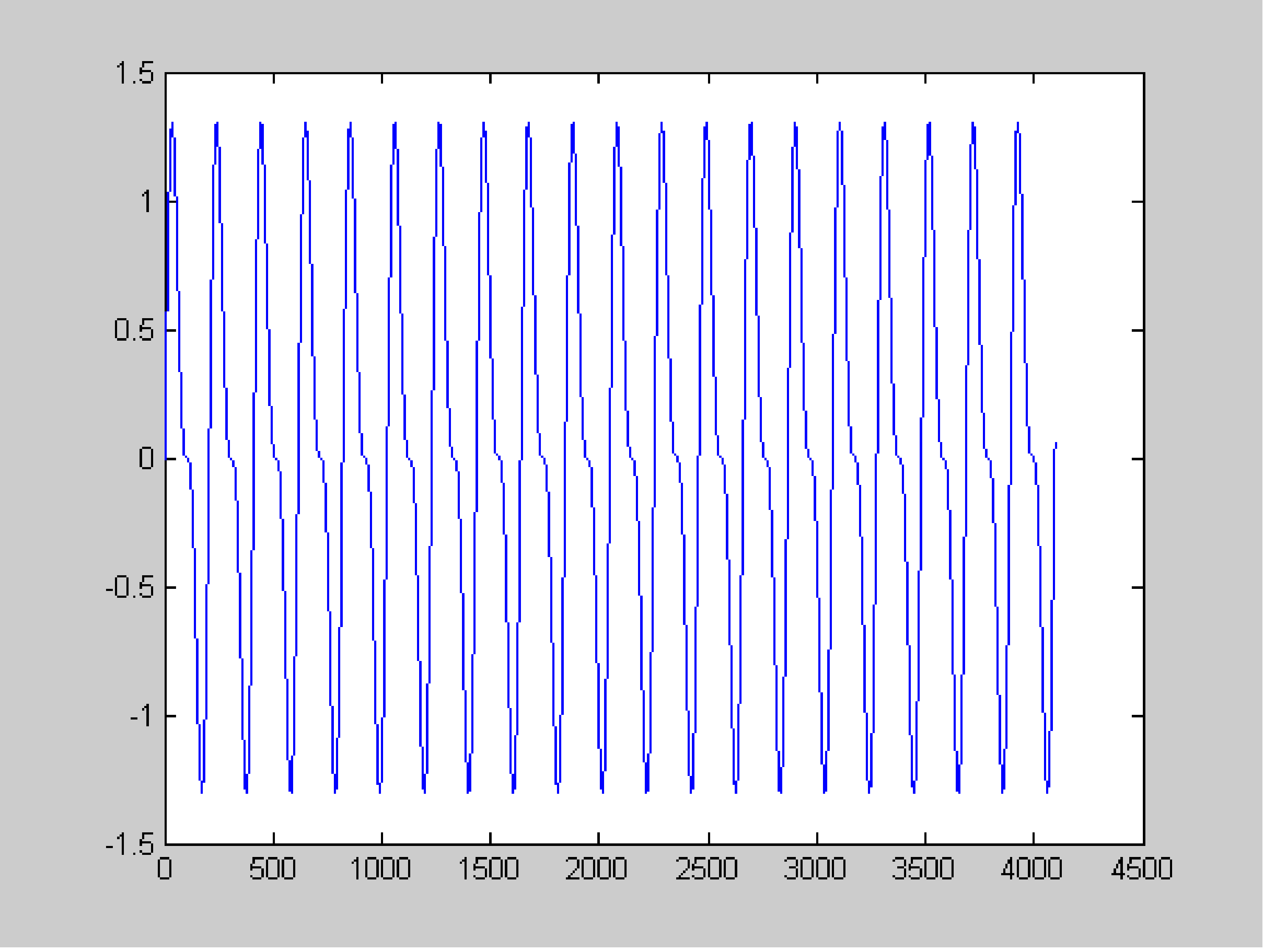
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

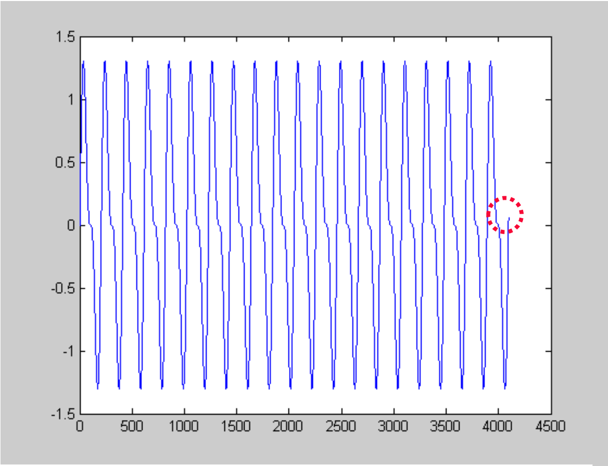
Consider N_P=20.01 N=4096

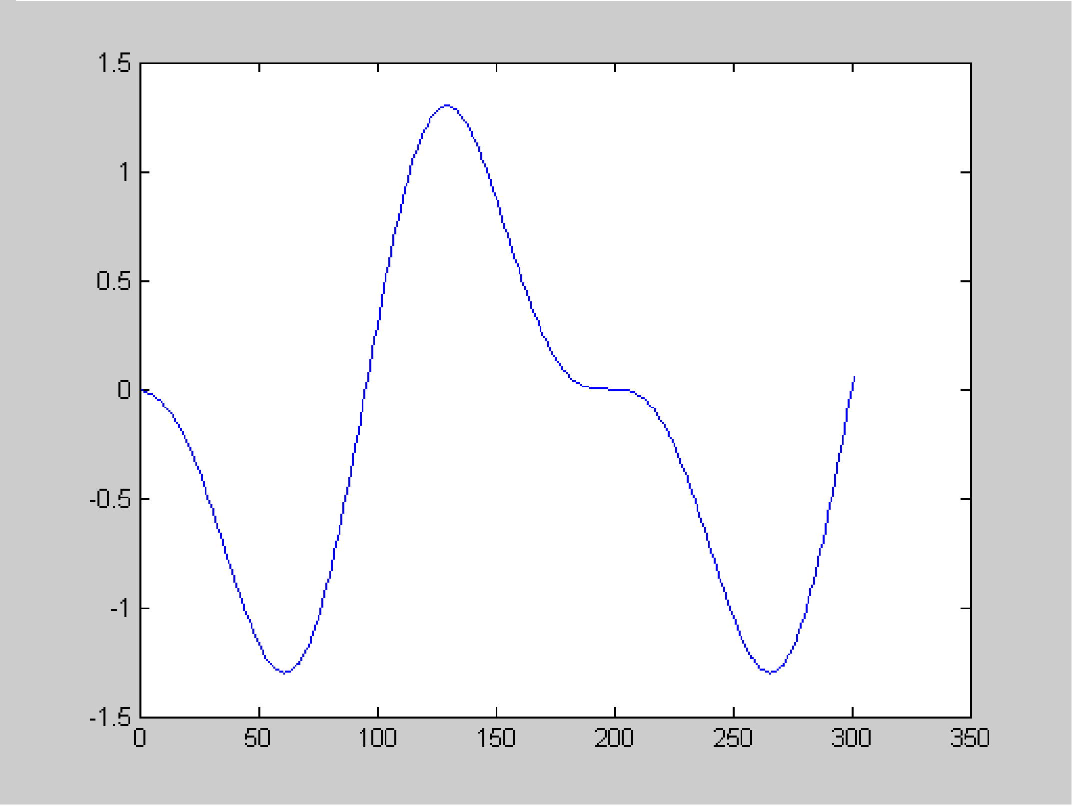
Deviation from hypothesis is .05% of the sampling window

Input Waveform

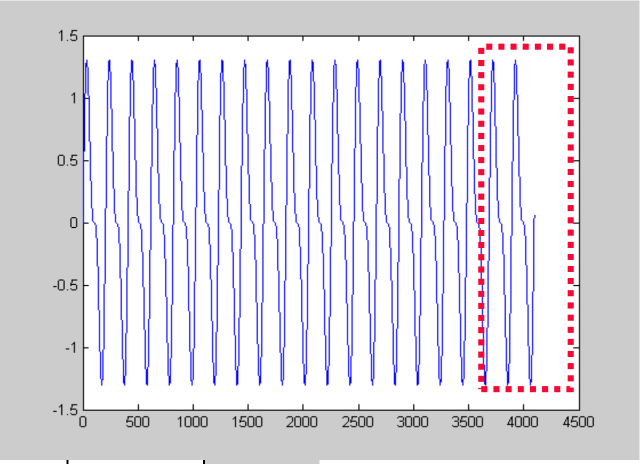


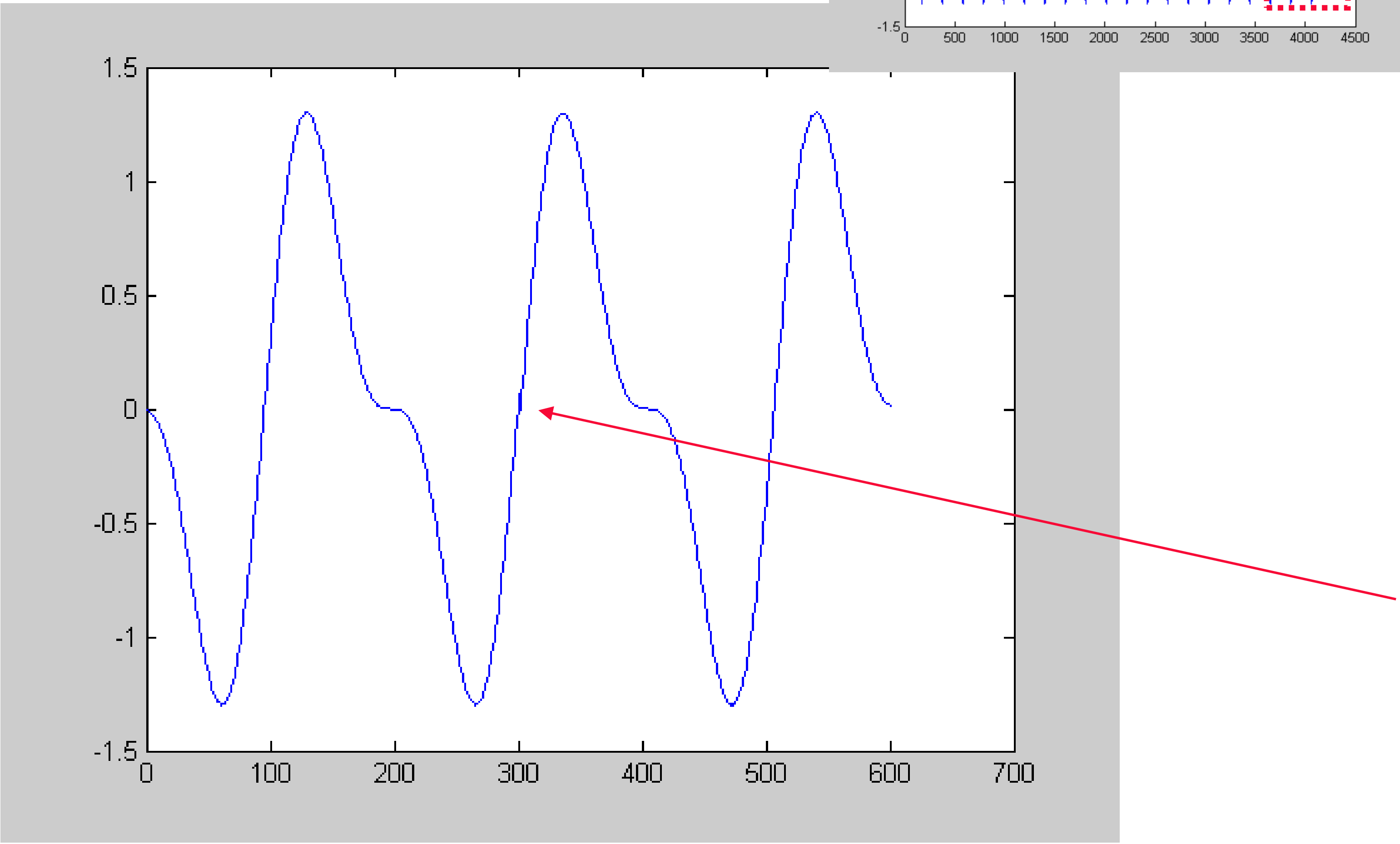
Input Waveform



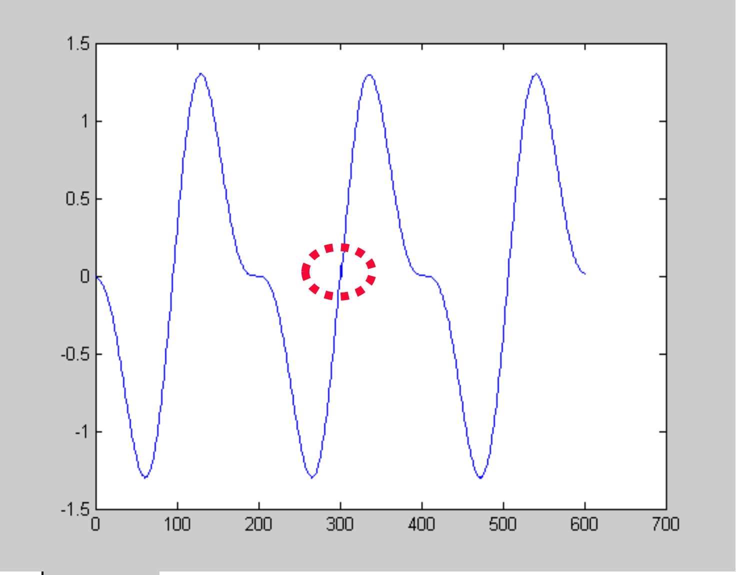


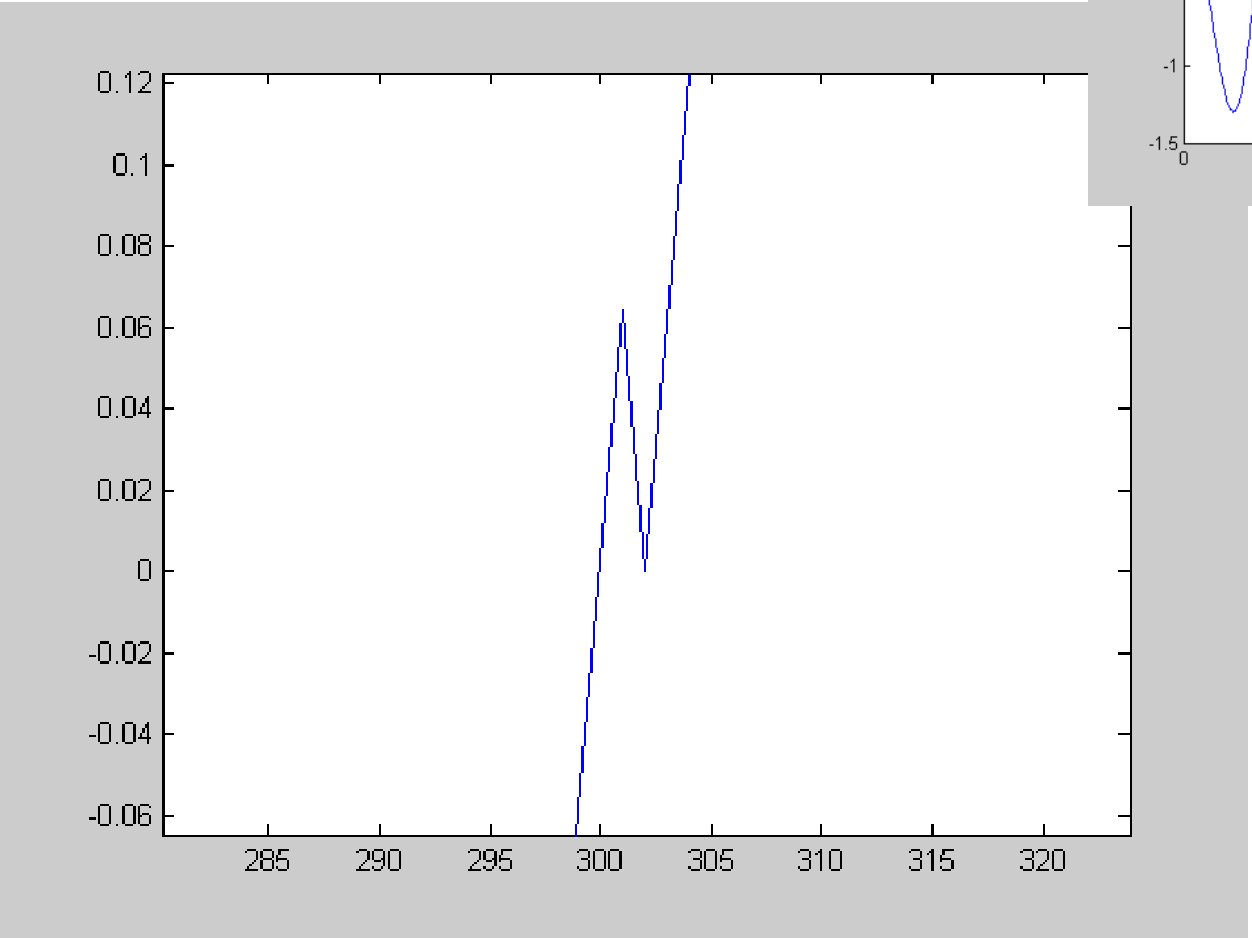
Input Waveform



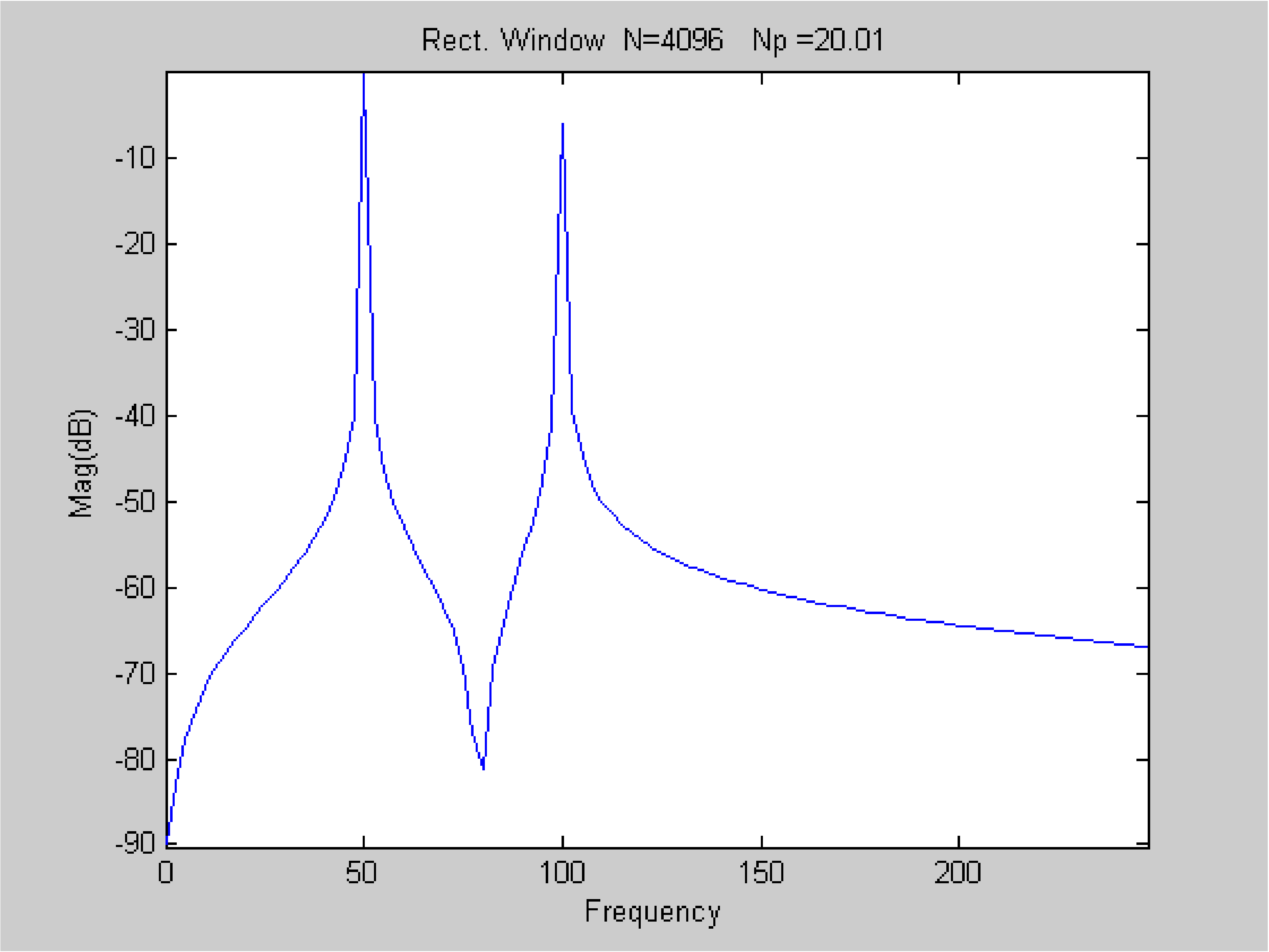


Input Waveform





Spectral Response with Non-Coherent Sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-89.8679 -83.0583 -77.7239 -74.2607 -71.6830 -69.5948 -67.8044

Columns 8 through 14

-66.2037 -64.7240 -63.3167 -61.9435 -60.5707 -59.1642 -57.6859

Columns 15 through 21

-56.0866 -54.2966 -52.2035 -49.6015 -46.0326 -40.0441 -0.0007

Columns 22 through 28

-40.0162 -46.2516 -50.0399 -52.8973 -55.3185 -57.5543 -59.7864

Columns 29 through 35

-62.2078 -65.1175 -69.1845 -76.9560 -81.1539 -69.6230 -64.0636

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 36 through 42

-59.9172	-56.1859	-52.3380	-47.7624	-40.9389	-6.0401	-39.2033

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB)
- Errors at about the 6-bit level !

Example

WLOG assume f_{SIG}=50Hz

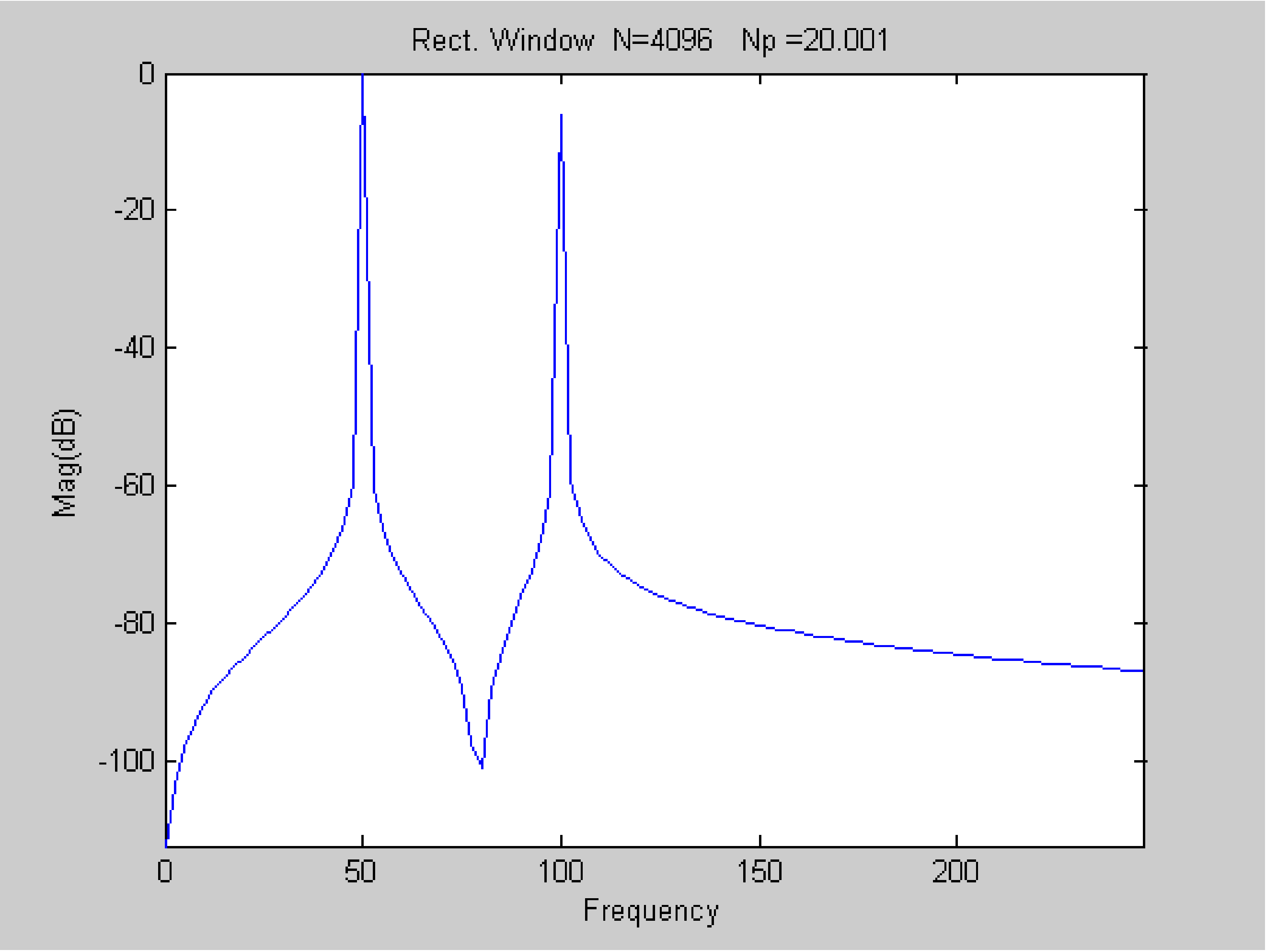
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.001 N=4096

Deviation from hypothesis is .005% of the sampling window

Spectral Response with Non-coherent Sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-112.2531 -103.4507 -97.8283 -94.3021 -91.7015 -89.6024 -87.8059

Columns 8 through 14

-86.2014 -84.7190 -83.3097 -81.9349 -80.5605 -79.1526 -77.6726

Columns 15 through 21

-76.0714 -74.2787 -72.1818 -69.5735 -65.9919 -59.9650 0.0001

Columns 22 through 28

-60.0947 -66.2917 -70.0681 -72.9207 -75.3402 -77.5767 -79.8121

Columns 29 through 35

-82.2405 -85.1651 -89.2710 -97.2462 -101.0487 -89.5195 -83.9851

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-79.8472 -76.1160 -72.2601 -67.6621 -60.7642 -6.0220 -59.3448

Columns 43 through 49

-64.8177 -67.8520 -69.9156 -71.4625 -72.6918 -73.7078 -74.5718

Columns 50 through 56

-75.3225 -75.9857 -76.5796 -77.1173 -77.6087 -78.0613 -78.4809

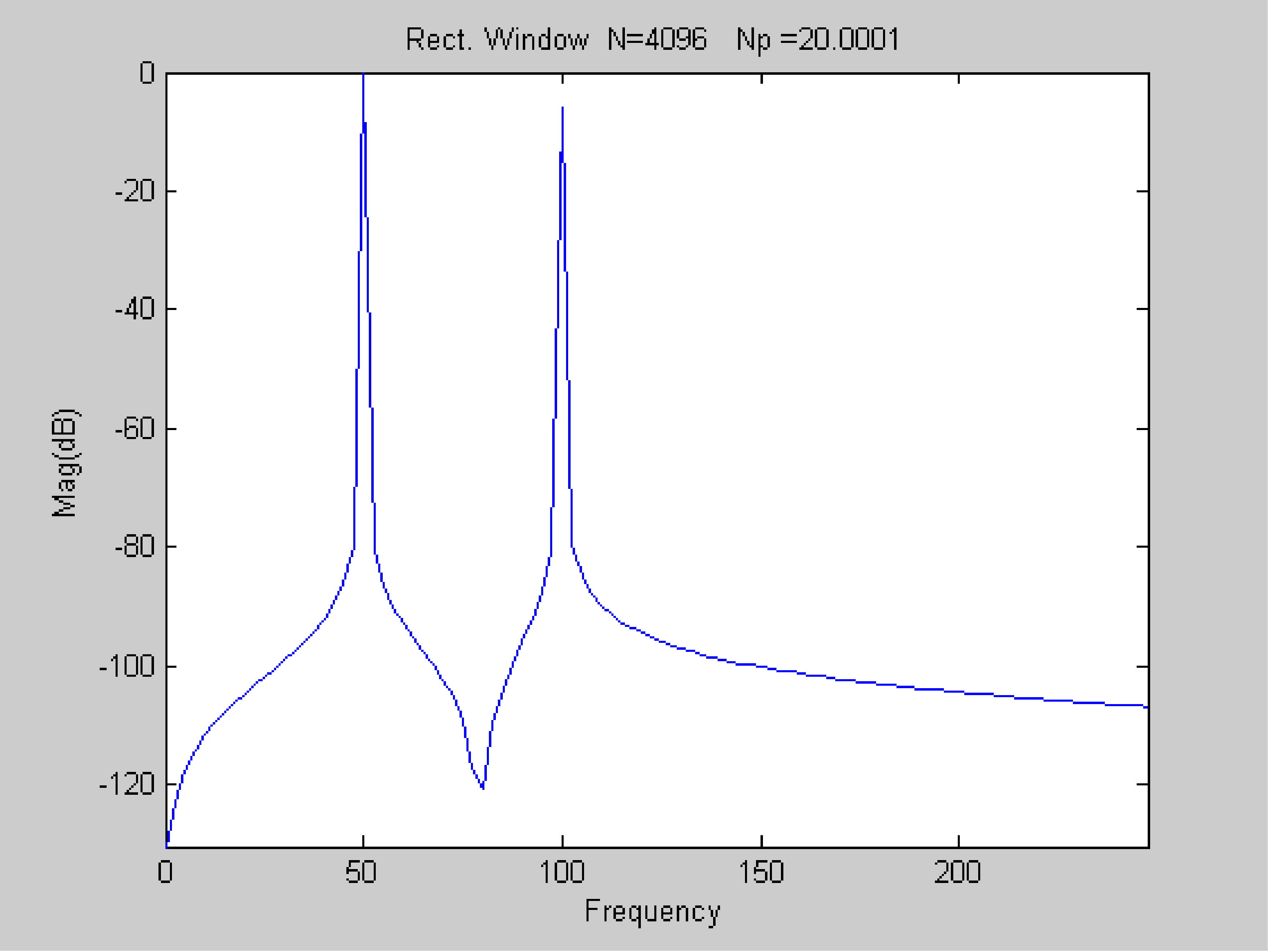
Columns 57 through 63

-78.8721 -79.2387 -79.5837 -79.9096 -80.2186 -80.5125 -80.7927

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)
- Errors at about the 10-bit level !

Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965

Columns 8 through 14

-106.1944 -104.7137 -103.3055 -101.9314 -100.5575 -99.1499 -97.6702

Columns 15 through 21

-96.0691 -94.2764 -92.1793 -89.5706 -85.9878 -79.9571 0.0000

Columns 22 through 28

-80.1027 -86.2959 -90.0712 -92.9232 -95.3425 -97.5788 -99.8141

Columns 29 through 35

-102.2424 -105.1665 -109.2693 -117.2013 -120.8396 -109.4934 -103.9724

k^{th} harmonic will appear at position 1+k•Np

Columns 36 through 42

-99.8382 -96.1082 -92.2521 -87.6522 -80.7470 -6.0207 -79.3595

Columns 43 through 49

-84.8247 -87.8566 -89.9190 -91.4652 -92.6940 -93.7098 -94.5736

Columns 50 through 56

-95.3241 -95.9872 -96.5810 -97.1187 -97.6100 -98.0625 -98.4821

Columns 57 through 63

-98.8732 -99.2398 -99.5847 -99.9107 -100.2197 -100.5135 -100.7937

Columns 64 through 70

Observations

- Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)
- Errors at about the 13-bit level !

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $$N > \frac{2f_{max}}{f_{SIGNAL}} N_P$$

Example $N < \frac{2f_{max}}{f_{SIGNAL}} N_P$ (Not meeting Nyquist sampling rate requirement)

If f_{SIG}=50Hz

and N_P=20 N=512

$$N > \frac{2f_{max}}{f_{SIGNAL}} N_P \Rightarrow f_{max} < 640Hz$$

Example

$$N < \frac{2f_{max}}{f_{SIGNAL}} N_P$$

(Not meeting Nyquist sampling rate requirement)

Consider N_P=20 N=512

If f_{SIG}=50Hz but an additional input at 700Hz is present

$$N_P = \frac{NT_S}{T} \leftrightarrow f_{SAMP} = f_{SIGNAL} \frac{N}{N_P} \qquad f_{SAMP} = 1.280KHz$$

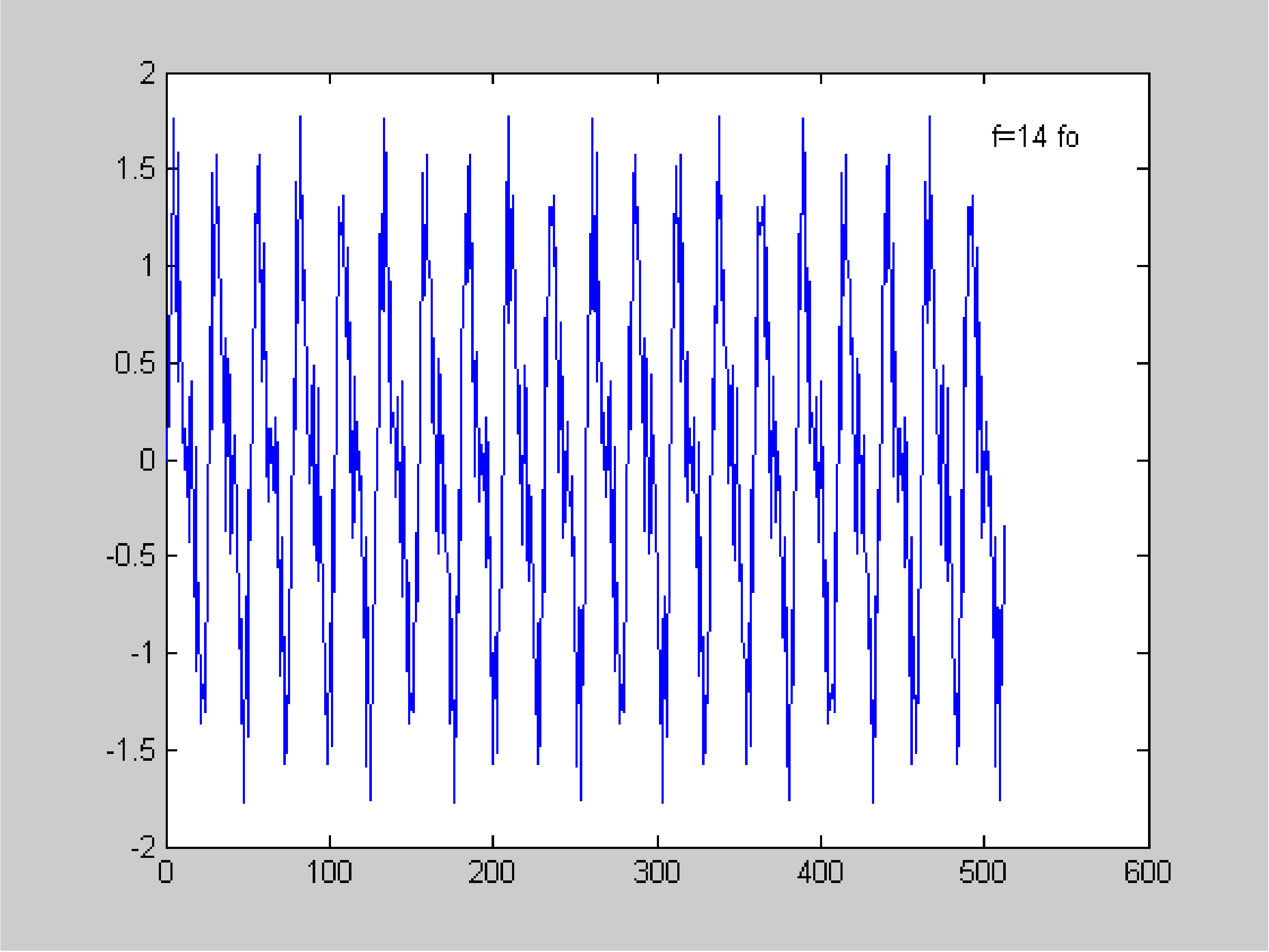
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t) + 0.5\sin(14\omega t)$$

$$\omega = 2\pi f_{SIG}$$

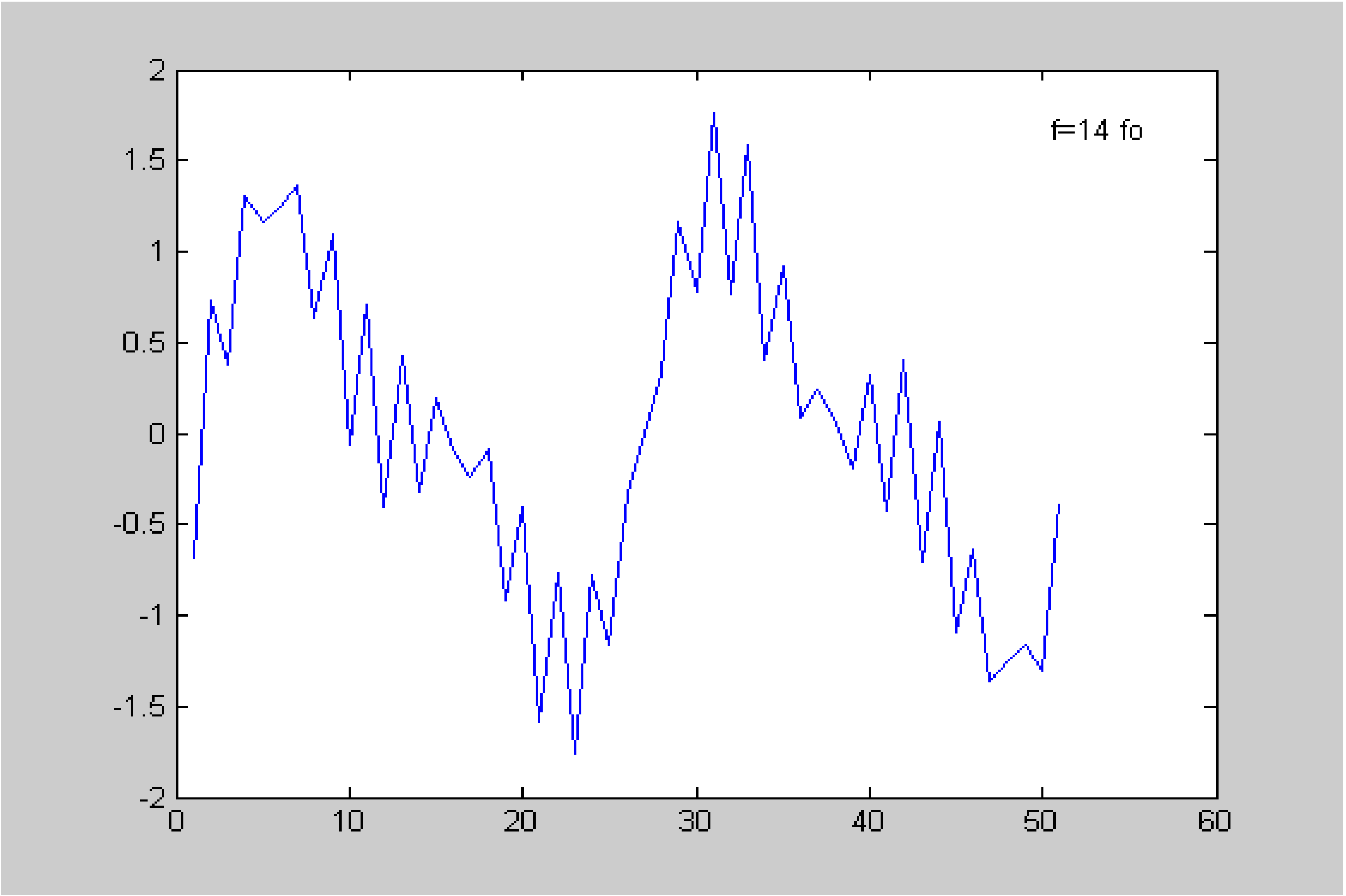
(i.e. the component at 700 Hz which violates the band limit requirement – Nyquist rate for the 700 Hz input is 14KHz)

Recall $20\log_{10}(0.5)=-6.0205999$

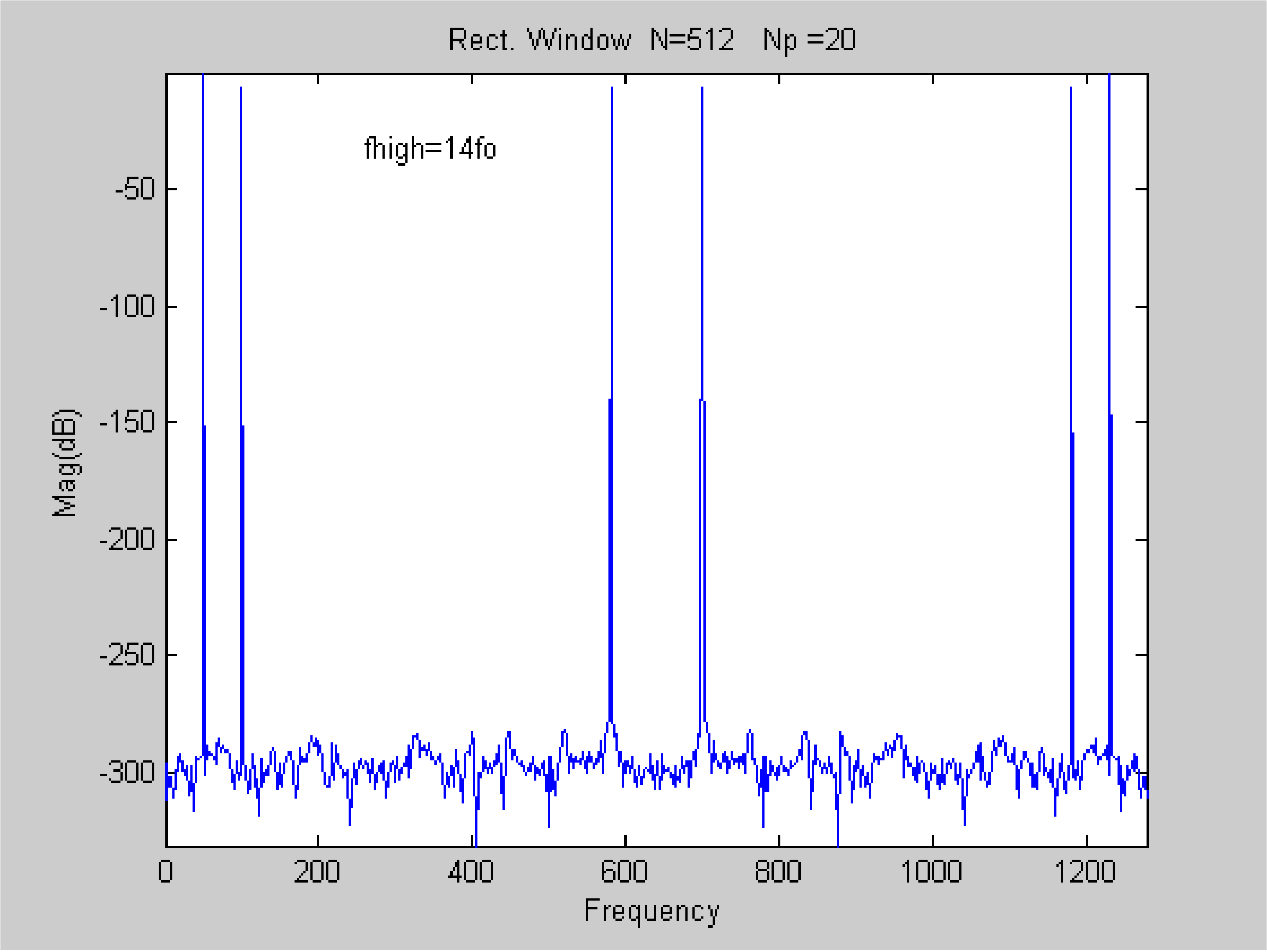
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

f_{high}=14fo

Columns 1 through 7

-296.9507 -311.9710 -302.4715 -302.1545 -310.8392 -304.5465 -293.9310

Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 -0.0000

Columns 22 through 28

-301.6889 -288.4812 -292.5621 -292.5853 -294.1383 -296.4034 -289.5216

Columns 29 through 35

-285.9204 -292.1676 -289.0633 -292.1318 -290.6342 -293.2538 -296.8434

Effects of High-Frequency Spectral Components

f_{high}=14fo

Columns 36 through 42

-301.7087 -307.2119 -295.1726 -303.4403 -301.6427 -6.0206 -295.3018

Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188

Effects of High-Frequency Spectral Components

Aliased components at

$$f_{alias} = f_{sample} - f$$

$$f_{alias} = 25.6f_{sig} - 14f_{sig} = 11.6f_{sig}$$

$$thus\ position\ in\ sequence = 1 + N_p \frac{f_{alias}}{f_{sig}} = 1 + 20 \bullet 11.6 = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238

-273.9840 -6.0206 -274.2295 -284.4608 -283.5228 -297.6724 -291.7545

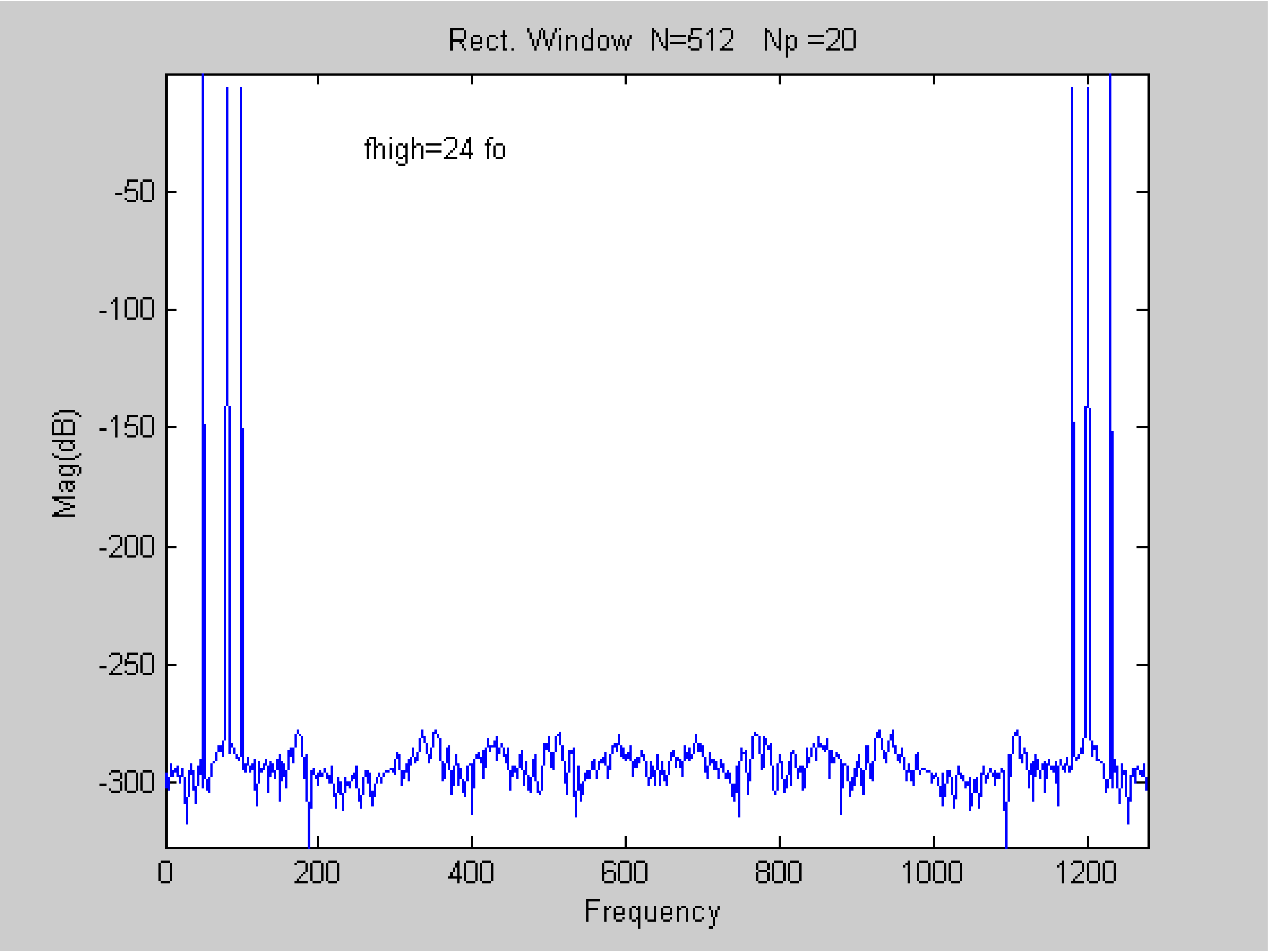
Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

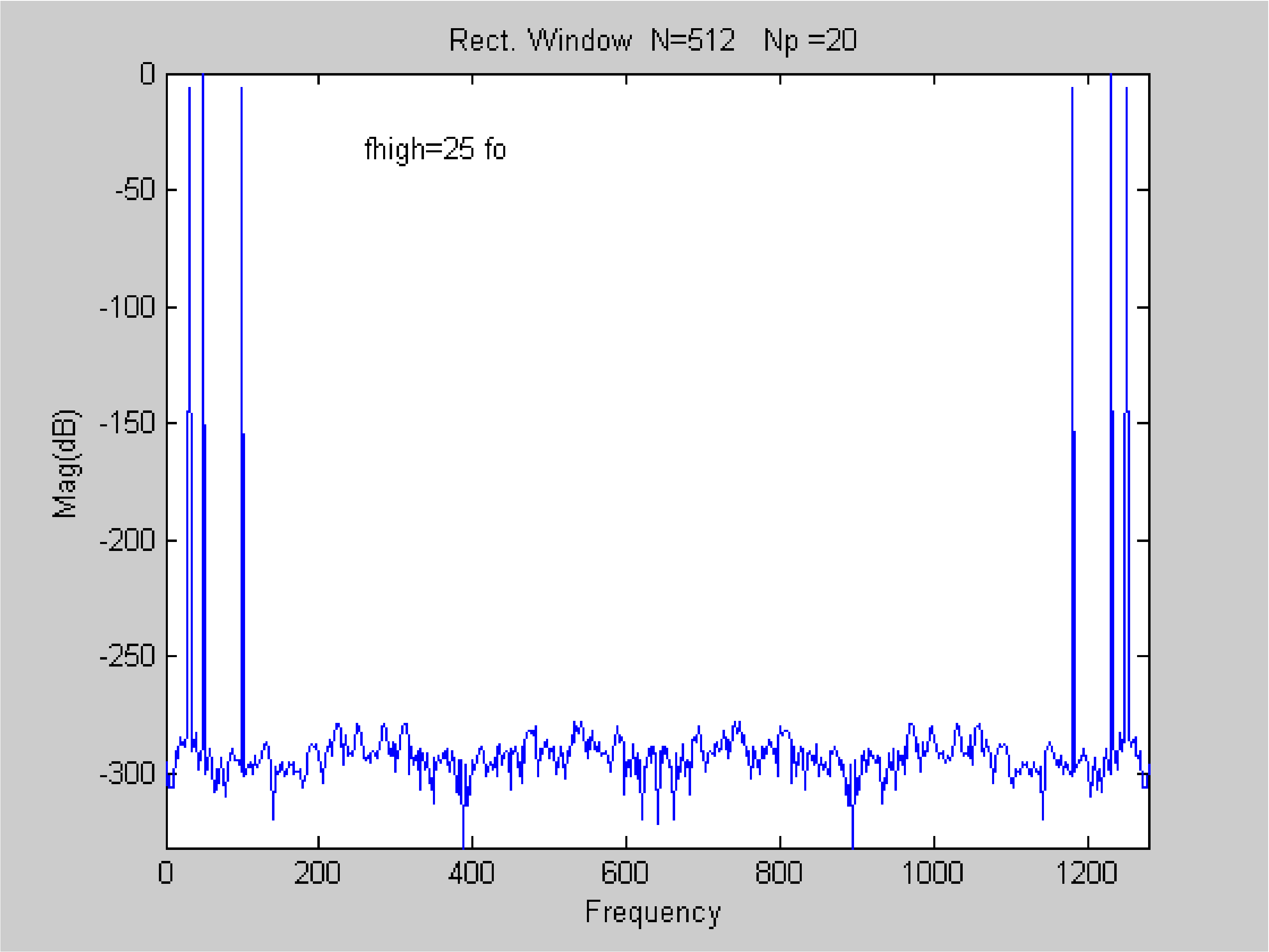
Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956

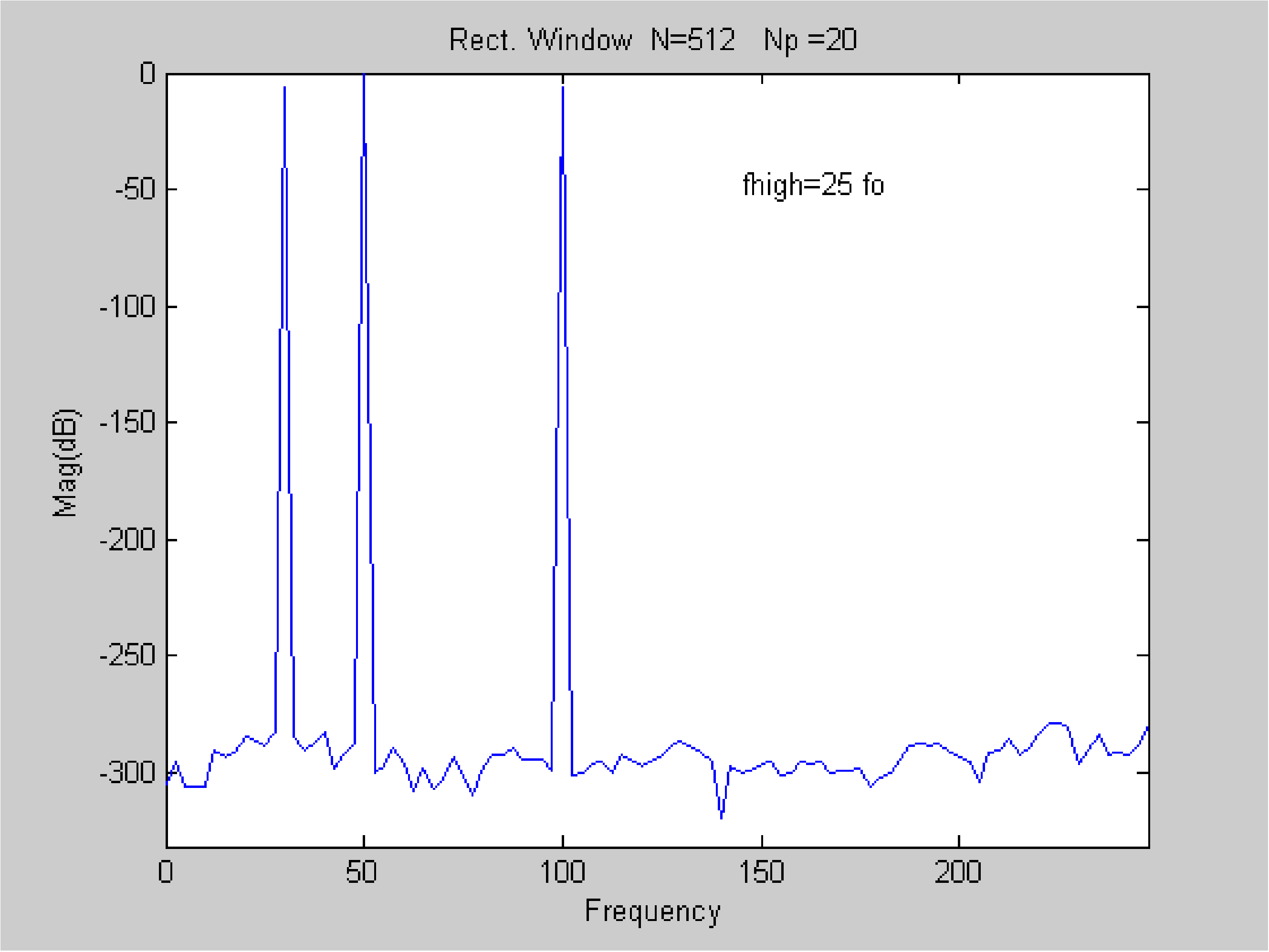
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

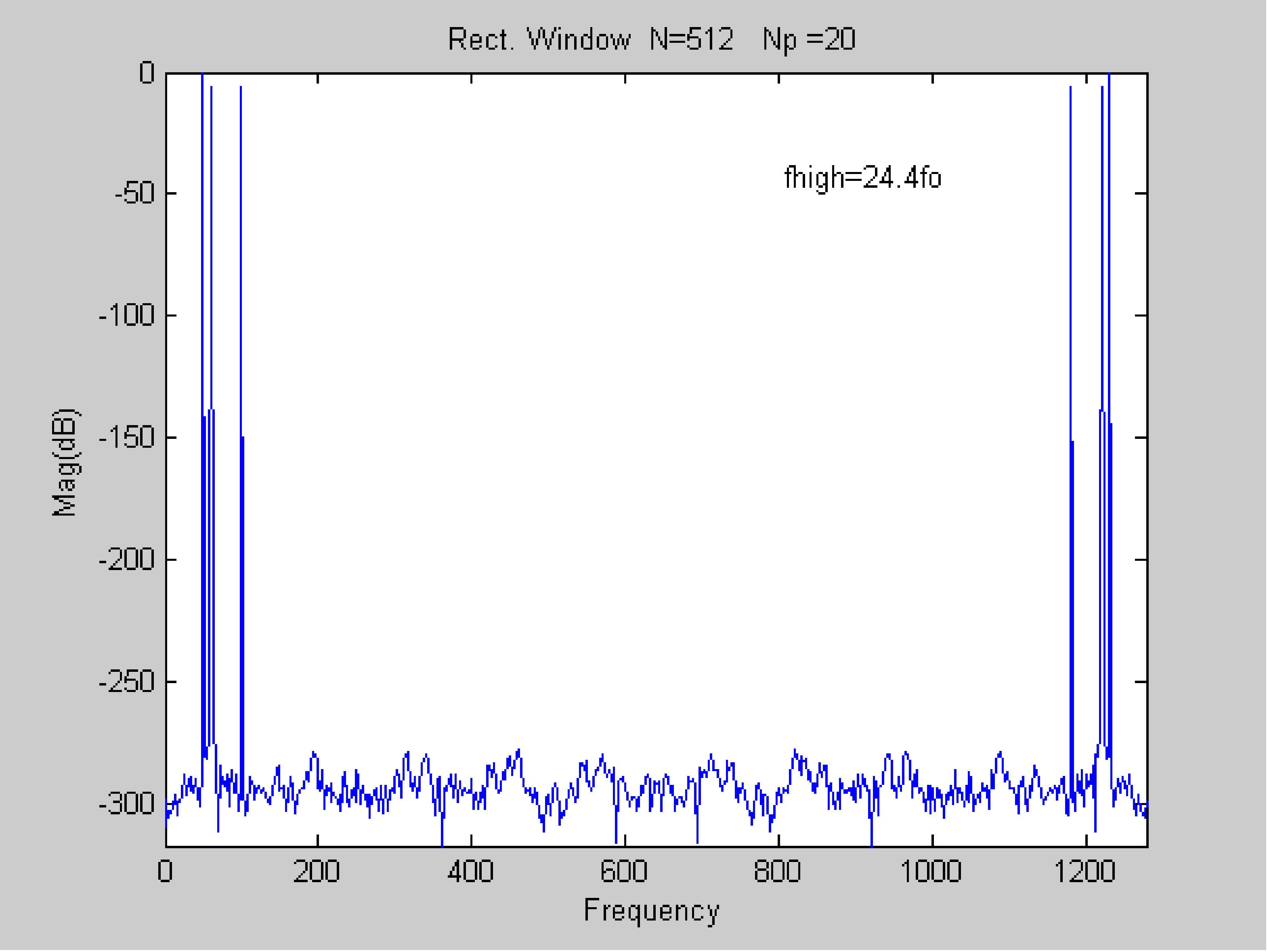


Effects of High-Frequency Spectral Components

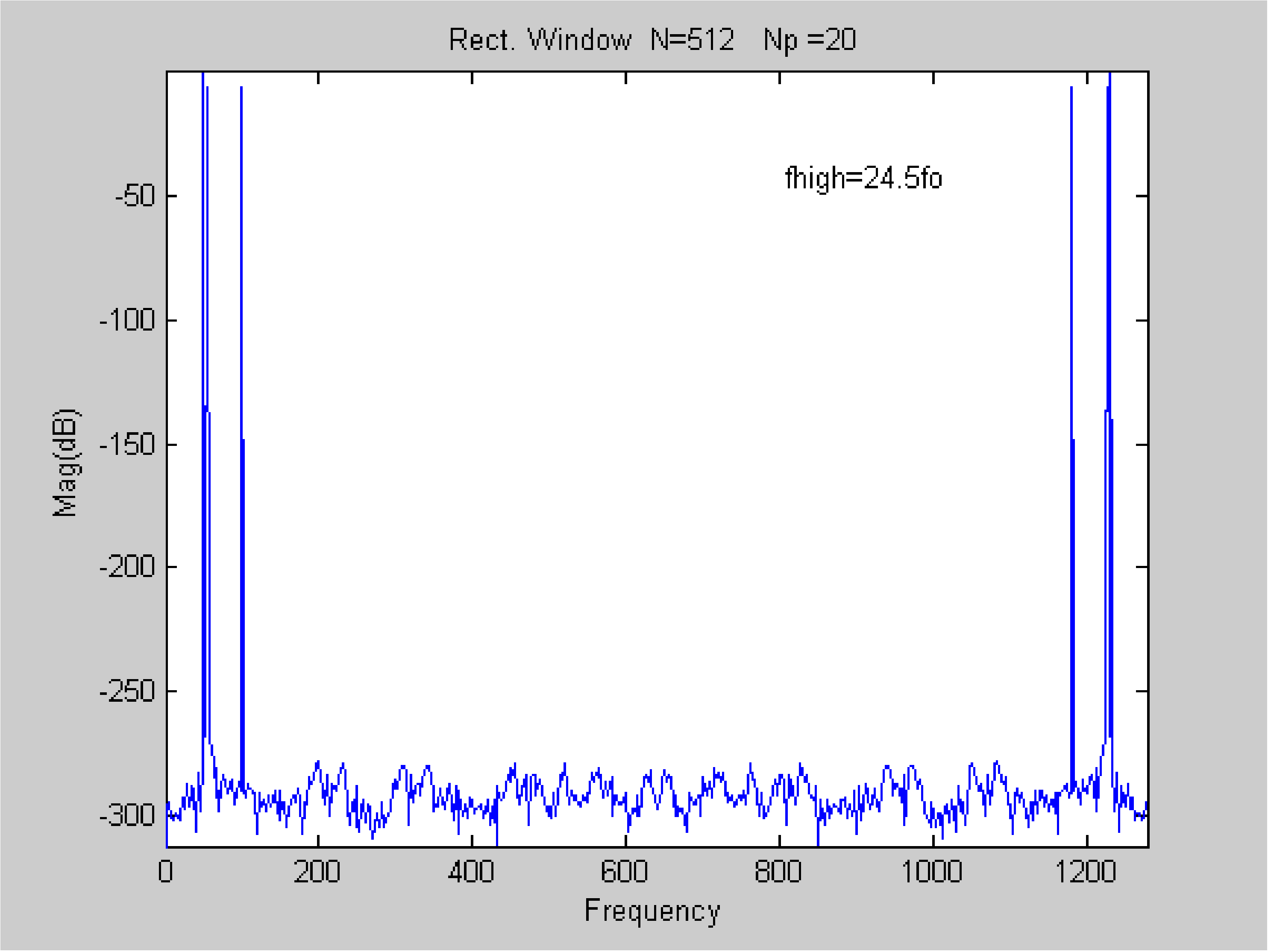


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Review Questions

Q1: How many DFT terms are there if the sample window is of length 4096?

A: 4096

Q2: When the magnitude of the DFT coefficients are plotted, the horizontal axis is an index axis (i.e. dimensionless) but often the index terms are labeled as frequency terms. If the sampling frequency is f_s and N samples are taken, what is the frequency of the first DFT term? What is the frequency of the 2^{nd} DFT term?

A: 0 Hz   A: fs/N

Q3: If samples of the time-domain signal are made over exactly 31 periods, which index term corresponds to the fundamental? To the second harmonic?

A: 32^{nd} term   A: 63^{rd} term

Q4: What is the difference between the DFT and the FFT**?**

A: FFT is a computationally efficient method of computing the DFT

Q5: True or False: The DFT terms are real numbers.

A: False   We are, however, often interested most in the magnitude of the DFT terms and these are real

Q6: True or False: The magnitude of the DFT terms are symmetric around index number N/2.   A: Yes

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

Stay Safe and Stay Healthy !

End of Lecture 28